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DETERMINATION OF THE COSMOLOGICAL RATE OF CHANGE OF G AND THE TIDAL ACCELERATIONS OF EARTH AND MOON FROM ANCIENT AND MODERN ASTRONOMICAL DATA

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PREFACE

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ABSTRACT

This study undertakes to improve the theory and numerical analysis of the ancient astronomical observations (−1374 to 1715), and to combine these with the modern data in a simultaneous solution for: $\dot{n}_t$ the tidal acceleration of the lunar longitude; $\dot{\omega}/\omega$ the observed apparent acceleration of the earth's rotation; $\dot{\omega}/\omega_{TNT}$ the true nontidal geophysical part of $\dot{\omega}/\omega$; and $\dot{G}/G$ the rate of change in the Gravitational Constant.

Error in the lunar node is shown to have a large systematic effect on $\dot{n}_t$ from eclipses, and corrections of +4.39 "cy^{-1} to $\dot{\Omega}$ and −0.10 "cy^{-2} to $\ddot{\Omega}$ are added to the standard ephemerides based on the results of other studies. The complete formulation for $\Delta T = A + B \cdot T + C \cdot T^2$ is essential to finding $\dot{\omega}/\omega$, and in testing the constancy of $\dot{\omega}/\omega$ and $\dot{n}_t$ during the historical period. An exact linear inequalities statistical processing of the large solar eclipses is substituted for traditional least squares in order to preserve observational sensitivity in the separation of $\dot{\omega}/\omega$ and $\dot{n}_t$. A strictly observational separation of $\dot{\omega}/\omega_{TNT}$ and $\dot{G}/G$ is made by including ancient data on Universal Time (UT) and very modern data on Atomic Time (AT) in the solution.

The ancient eclipses alone provide $\dot{n}_t = -30.0 \pm 3.0$ "cy^{-2} and the corresponding $\dot{\omega}/\omega = -24.3 \pm 2.0 \times 10^{-11}\,yr^{-1}$, and $\Delta T = 20 + 114 \cdot T + 38.30 \cdot T^2$ (T in cy from 1900). This $\dot{\omega}/\omega$ agrees very well with $\dot{\omega}/\omega$ from timed solar equinoxes alone (independent of the moon): $\dot{\omega}/\omega = -23.8 \pm 2.3 \times 10^{-11}\,yr^{-1}$, and gives an independent verification that the $\dot{n}_t$ from eclipses is not significantly biased. The observational bounds imposed on ΔT as a function of time prove that the accelerations have remained constant during the historical period within uncertainties. These three results from the ancient observations are then combined with: (i) $\dot{n}_t = -26 \pm 2$ "cy^{-2} from transits of Mercury (1677 − 1975) and (ii) $\dot{n}_a = -36 \pm 5$ "cy^{-2} from lunar occultations on Atomic Time, provided by other author's studies. The simultaneous solution from all data provides $\dot{n}_t = -27.2 \pm 1.7$ "cy^{-2}, $\dot{\omega}/\omega = -22.6 \pm 1.1 \times 10^{-11}\,yr^{-1}$, $\Delta T = 20 + 114 \cdot T + 35.55 \cdot T^2$; where, on the so-called primitive cosmologies (*e.g.* Hoyle-Narlikar), $\dot{G}/G = -2.3 \pm 1.5 \times 10^{-11}\,yr^{-1}$ with $\dot{\omega}/\omega_{TNT}$ between +3.1 and +5.9 $\pm \sim 4 \times 10^{-11}\,yr^{-1}$ depending upon the assumed earth expansion factor as a function of $\dot{G}/G$; or on the Dirac cosmology, $\dot{G}/G = -5.1 \pm 3.0 \times 10^{-11}\,yr^{-1}$ and $\dot{\omega}/\omega_{TNT} = +3.1 \pm 4.4 \times 10^{-11}\,yr^{-1}$; or constraining $\dot{G}/G = 0$, $\dot{\omega}/\omega_{TNT} = +9.2 \pm 2.5 \times 10^{-11}\,yr^{-1}$. The Brans-Dicke cosmology is consistent with a "primitive" $\dot{G}/G$ as found above (near $-2.3 \times 10^{-11}\,yr^{-1}$). In the remaining cosmologies, a Hubble Constant of 55 ± 7 km/s/Mpc corresponds to $\dot{G}/G = -5.6 \pm 0.7 \times 10^{-11}\,yr^{-1}$. The Dirac cosmology agrees well with this result from extragalactic data, the others needing more $\dot{\omega}/\omega_{TNT}$ to reveal $\dot{G}/G$ than is available. VanFlandern finds $\dot{G}/G$ from data sets (i) and (ii) above: $\dot{G}/G = -5.8 \pm 3.0 \times 10^{-11}\,yr^{-1}$. Solving for $\dot{G}/G$ on any cosmology above gives $\dot{\omega}/\omega_{TNT}$ consistent with zero. Several *possible* (but unproven) geophysical theories have been advanced to explain a $\dot{\omega}/\omega_{TNT}$ in the range found above, including post glacial uplift, geomagnetic torques, and phase-change core. Assuming $\dot{\omega}/\omega_{TNT} = 0$ with the timed ancient equinoxes' $\dot{\omega}/\omega$ above, independent of (ii) and the moon, and data set (i), provides an estimate of $\dot{G}/G = -6.9 \pm 3.0 \times 10^{-11}\,yr^{-1}$ independent of VanFlandern's Atomic Time data and consistent with the other determinations. It appears that either we really have a cosmological $\dot{G}/G$ consistent with the Hubble Constant, or we have a significant $\dot{\omega}/\omega_{TNT}$. The former is consistent with all observations and the relationships between $\dot{G}/G$ and the Hubble Constant contained in several modern cosmologies; the latter can be forced only if the data (ii) is wrong *and* we abandon the Hubble Constant versus $\dot{G}/G$ relationship common to modern cosmologies. Final $\dot{\omega}/\omega_{TNT}$ with Hubble rate on Dirac is: $+2.5 \pm 2.5 \times 10^{-11}\,yr^{-1}$.

This provides three *independent* determinations of a rate of change of G consistent with the Hubble Constant and a near-zero nontidal rotational acceleration of the earth, $\dot{\omega}/\omega_{TNT}$. The tidal accelerations are shown to have remained constant during the historical period within uncertainties. Ancient and modern solar system data, and extragalactic observations provide a completely consistent astronomical and cosmological scheme.

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Background and Introduction

This paper is an updated summary of the principal results of Muller (1975). The study emphasizes the theory and numerical analysis of the ancient astronomical observations, taking the historical and linguistic background from other authors. The strongest data are the large solar eclipses. The lunar parallax (distance) was determined from such an observation by Hipparchus (*ca.* 135BC). Sir Edmund Halley (1695) was the first modern analyst to use ancient eclipses, and he discovered the so-called secular acceleration of the moon's longitude. His discovery was confirmed and refined by Dunthorne (1749), Baily (1811), Airy (1857) and Newcomb (1875) among others before the present century.

Euler (1770) won the prize of the French Academy for a *theoretical* explanation of the secular acceleration with a proof that it could not come from any defect in lunar theory. Lagrange (1774) proved that the figure of the earth, and hence the planetary terms, could not be responsible. Laplace (1786) found a large theoretical contribution from the changing eccentricity of the earth's orbit, proving that Euler had erred, but this accounted for only part of the discrepancy. The modern lunar theory was created by Hansen (1857) and Brown (1919).

All this time it was assumed that the secular acceleration of the moon was *real* as opposed to *apparent*. Spencer Jones (1939) proved that part of the acceleration was apparent only, arising from the slowing of the earth's rotation. The lunar longitude, and earth's rotation, were both accelerated by the effects of tidal action as suggested by Immanuel Kant (1754) nearly two centuries earlier! Kant made the astounding argument that the earth's rotation rate could *not* be constant because the tidal drag must *inevitably* slow it down! Kelvin (1897) notes that nobody anticipated Kant in this fundamental insight, and Hastie (1900) provides a translation of Kant's treatise with a valuable commentary. Amazingly, Kant later (in his *Physical Geography*) withdrew the argument, and suggested that this effect might be offset by the accretion of the earth's core! Urey (1952) considers a similar hypothesis, and it is still being debated by geophysicists, along with other possible sources of change in the earth's moment of inertia (as below).

The so-called Spencer Jones anomaly emerged as modern studies yielded values of the lunar tidal acceleration $\dot{n}_t$ from ancient observations (top of Figure 1) and modern observations (second block of Figure 1), which differed substantially. Newton (1970) argued from his data analysis that the lunar tidal acceleration had changed during historical time, but this view conflicts with theory as noted by Munk & MacDonald (1960) and with the data analysis in this paper. Both the lunar longitude and earth's rotation are accelerated by the tidal couple, and both can be independently determined from the observations in the manner described below. Since angular momentum is conserved, we can check the observational determinations for consistency. Results of previous studies, *e.g.* Newton (1970), indicated a very large residual nontidal acceleration in the earth's rotation $\dot{\omega}/\omega_{NT} = 23 \times 10^{-11}\,yr^{-1}$. This must arise from changes in the moment of inertia or interchange of angular momentum inside the earth, or a rate of change in the Gravitational Constant as first suggested by Dicke (1957). The present research was primarily motivated by the existence of these two long-standing unexplained anomalies.

Figure 1

Previous Determinations of Lunar Acceleration

Author	Epoch	$\dot{n}$	$\sigma\dot{n}$
Fotheringham (1920)	-200	-30.8†	
De Sitter (1927)		-37.7	4.3
Newton (1970) All Ancient Data	-200	-41.6	4.3
Newton (1970) All Medieval Data	1000	-42.3	6.1
Stephenson (1972)	-300	-34.2	1.9
Muller & Stephenson (1975) ILE node	-400	-37.5	5.0
Muller (1975) Ancient eclipses, ILE	-470	-34.5	3.0
This Paper: Eclipses, corrected node	-470	-30.0	3.0
Spencer Jones (1939)	1800	-22.4	1.1
Clemence (1948)		-17.9	4.3
VanFlandern (1976)	1965	-35.0	5.0§
Oesterwinter & Cohen (1972)	1940	-38.0	8.0¶
Newton (1968) Satellite Data	1965	-20.1	2.6
Morrison & Ward (1975)	1830	-26.0	2.0
Lambeck (1975) From Tidal Theory		-35.0	4.0
		$''cy^{-2}$	

† Newton (1970) finds the corrected value -34.0

§ On Atomic Time and will include effects of $\dot{G}/G$

¶ VanFlandern (private communication) prefers range -18 to $-38''cy^{-2}$; some of the data are on Atomic Time.

Theoretical Foundation

We are engaged here in a comparison of lunar (and solar) orbital theory with observations of the sun and moon. The reference lunar ephemeris is the Improved Lunar Ephemeris (1954), often denoted "ILE". The theory of the sun is that of Newcomb (1895). Corrections derived in this study are to be added to the *mean* elements, nominal values of which may also be found in the Explanatory Supplement to the Astronomical Ephemeris (1961). The theories are constructed in such a way that if you desire a position, you begin with the mean values, and then

add the theoretical periodic *perturbations*. The expression for the mean lunar longitude λ_m is:

$$\lambda_m = A_o + B_o \cdot T + (C_o + C_t) \cdot T^2 + D_t \cdot T^3 \quad (1)$$

where T is expressed in Julian centuries from 1900, 0 January, 12 hours ET (Ephemeris Time). For the lunar node:

$$\Omega = E_o + F_o \cdot T + G_t \cdot T^2 + H_t \cdot T^3 \quad (2)$$

For the mean solar longitude λ_s:

$$\lambda_s = I_o + J_o \cdot T + K_t \cdot T^2 + L_t \cdot T^3 \quad (3)$$

The letters A through L represent the ephemeris coefficients for the mean positions at *epoch* T, and the subscripts "o" and "t" denote observationally and theoretically determined values respectively. It is clear that A, B, E, I and J must be determined by observation since these are the positions and rates of the parameters at the epoch (which is quite arbitrary). The laws of Newton relate B and J to quantities such as masses of the primary bodies and the distances of the orbiting bodies, but these must be observed. If there were no tides acting on the earth-moon system, then the coefficient C would be strictly determined theoretically from Newton's theory. The tidal part must be observed as an acceleration in lunar longitude, C_o, and adds to the theoretically computed part C_t. That determination is central to this analysis and many before it (figure 1). The earth's angular momentum about the sun is so large that tidal effects on coefficient K are negligible.

Since "n" is the standard symbol for the lunar (or planetary) rate in longitude, $\dot{n}$ is the acceleration which we must observe, subscripted "t" for the true tidal part, and "a" if observed on Atomic Time instead of Ephemeris Time, where:

$$C_o = \tfrac{1}{2}\dot{n}_t \quad (4)$$

As first noted in Dicke (1957), these two time scales will differ if the Gravitational Constant or an equivalent physical parameter is changing, rather than remaining constant as assumed in Newtonian mechanics. This is considered further below.

Problem of the Lunar Node

In principle, F_o (2) should be theoretically calculable from Newtonian mechanics, given the masses, orbits and figures of the earth, moon, sun and major planets. As noted in Eckert (1965) and elsewhere, this has not been a practical undertaking. An observational determination due to Brown (1914) and reconsidered by Spencer Jones (1932) was used in the ILE (1954). Muller, Newhall, VanFlandern & Williams (1976) have very recently run a computer processed numerical integration of the solar system including the moon back more than 250 years.

Although initial orbital conditions and physical constants such as masses and distances had to be observationally determined in other research, primarily lunar, planetary and spacecraft laser and radio ranging, it is only necessary to have a reasonably converged integration to determine the lunar nodal rate. The result is given here as the correction to be added to the nominal ILE (1954) coefficient F_o:

$$\dot{\Omega} = +4.39 \pm 0.15 \text{ "cy}^{-1} \quad (5)$$

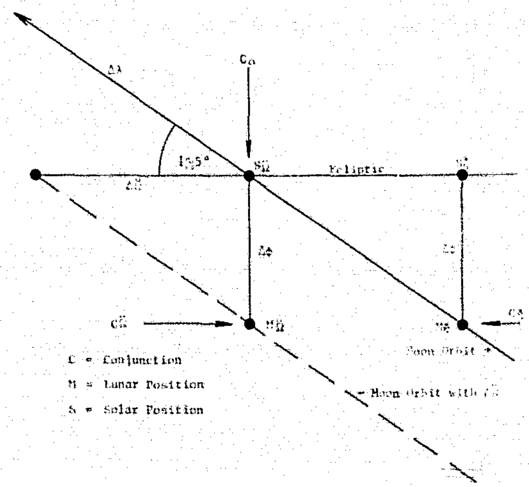
The formal uncertainty is provisional and is still being studied as work on the referenced paper is completed. This result agrees very well with the observational value of Martin & VanFlandern (1970) of $\dot{\Omega} = +4.31 \pm 0.36$ "cy^{-1}, which VanFlandern (private communication) had worried might be adversely affected by systematic errors in the observations; and with Muller (1975) from a combination of ancient and modern observations similar to that undertaken below, of $\dot{\Omega} = +7.1 \pm 3.4$ "cy^{-1} (assuming $\dot{G}/G = 0$), and $+8.2 \pm 3.6$ "cy^{-1} (solving for $\dot{G}/G$). The correction in (5) will therefore be made to the ephemeris (or results) used in the reduction of the ancient eclipses.

Lunar Node as a Systematic Error Source

Newton (1970), page 287, dismissed node as a possible *systematic* error in the analysis of ancient observations on the grounds that the small effects (5) average zero. His argument is sound for the *timed* observations, but Muller (1975) showed that it is incorrect for untimed data such as large solar eclipses (the dominant data source). Cowell (1905) had suggested that node errors would not be separable from the lunar longitude acceleration $\dot{n}$ in

Figure 2

Equivalence of Node & Longitude Change at Conjunction



ancient observations, but his point was overlooked in the studies, figure 1, before Muller (1975).

If the lunar orbit was exactly in the plane of the ecliptic, the eclipse observations would be unable to separate $\dot{n}$ and $\dot{\omega}/\omega$ (the earth's rotational acceleration). This can be seen in Figure 2 by noting: [i] $\dot{\omega}/\omega$ cannot be distinguished from an acceleration of the sun's longitude; [ii] neglecting lunar parallax change and other very small second-order effects arising from changing the sun's longitude, the acceleration of the moon cannot be distinguished from that of the sun if their orbits are in the *same plane*. The lunar inclination I of about 5° provides latitude variation $\Delta\phi$ at conjunction which maps to the earth's surface in geometric eclipse limits (for example). The altered *geometry* provides the small, but essential, separation of $\dot{n}$ and $\dot{\omega}/\omega$.

For a nominal conjunction at the node C_o in figure 2, a change in $\dot{n}$ which would move the moon to the left of node an amount $\Delta\lambda$, as shown, gives rise to a new conjunction to the right of node at $C_{\dot{n}}$ due to the sun's mean motion n_s. This happens because the moon will reach the node sooner, and the sun will not yet have reached it. The sun is at $S_{\dot{n}}$ and the moon is in conjunction at $M_{\dot{n}}$, and :

$$\Delta\phi = \pm\ \Delta\lambda\ \left(\frac{n_s}{n_m \mathrm{Cos\,I} - n_s}\right)\ \mathrm{Tan\ I} \tag{6}$$

A direct motion of the node itself gives rise to a conjunction at $C_{\dot{\Omega}}$ with the sun at $S_{\dot{\Omega}}$ and the moon at $M_{\dot{\Omega}}$, and:

$$\Delta\phi = \pm\ \Delta\Omega\ \mathrm{Tan\ I} \tag{7}$$

where the (-) sign applies at the ascending node and the (+) sign at the descending node. The key point is that in *untimed* observations we can ignore the time difference implied by the sun's position at $S_{\dot{\Omega}}$ or $S_{\dot{n}}$ and changes arising from the inclination of the earth's equator, and moving the node and longitude *always* gives rise to the *same* geometric effect $\Delta\phi$ at each node. Furthermore, because of the sun's relatively small mean motion n_s compared with the moon's n_m, a change in the node is about 12 times as effective in changing $\Delta\phi$ as is a change in longitude ($\Delta\Omega = \Delta\lambda$):

$$\Delta\phi\ (\mathrm{from}\ \Delta\Omega) = 12.15\,\Delta\phi\ (\mathrm{from}\ \Delta\lambda) \tag{8}$$

There are other second-order variations which affect the projection of the $\Delta\phi$ at conjunction to the earth's surface in eclipses, but they are about a factor of five smaller than the basic *geometric* term in (8). A numerical test of this analysis can be easily made by inspecting the ratios of the partials with respect to $\dot{n}$ and $\ddot{\Omega}$ for each observation in Figure 9 below.

Muller (1975) calculated eclipses assuming $\dot{\Omega} = 0$, but anticipated that there would probably need to be a correction when a reliable $\dot{\Omega}$ became available. Since the apparently definitive result (5) became available only a few days before this writing, it will be simplest to use the calculations of Muller (1975) and apply a correction to $\dot{n}$ based on $\ddot{\Omega}$. The change in apparent $\Delta\dot{n}$ resulting from a change in ephemeris nodal acceleration $\Delta\ddot{\Omega}$ follows from (8):

$$\Delta\dot{n} = -12.15\Delta\ddot{\Omega} \tag{9}$$

For an equal $\Delta\Omega$ from rate and acceleration terms we have $\frac{1}{2}\ddot{\Omega}T^2 = \dot{\Omega}\,T$. Combining this with (9) yields:

$$\Delta\dot{n} = \{2(12.15)/T\}\,\dot{\Omega} = 24.3\,\dot{\Omega}\,/\,T \tag{10}$$

This gives the effect on $\dot{n}$ solved from eclipses at mean epoch T in the presence of an error in nodal rate $\dot{\Omega}$. The ancient eclipses in Muller (1975) which participate in the mean acceleration solutions range from the year 120 back to -1374, figure 14 below, and have a weighted mean epoch near the year -470. This epoch can be used with sufficint accuracy in the correction to $\dot{n}$ arising from $\dot{\Omega}$, and substituting $T = 23.7$ into (10) we have the change in apparent $\Delta\dot{n}$ resulting from a change in ephemeris nodal rate $\Delta\dot{\Omega}$:

$$\Delta\dot{n} = 1.02\,\mathrm{cy}^{-1}\Delta\dot{\Omega} \tag{11}$$

or for the correction implied by (5):

$$\Delta\dot{n} = +4.5\,''\mathrm{cy}^{-2} \tag{12}$$

which can be added to the uncorrected $\dot{n}$ found on the standard ephemeris as necessary below.

Muller (1975) undertook a reexamination of the basis for $\ddot{\Omega}$ in the analytical ephemeris. The coefficient G_t in (2) was computed by Brown (1919). Deprit & Henrard (1975) provide the partials relating various lunar parameters, which VanFlandern (private communication) has verified from the classical analysis of Delaunay. If the planetary or lunar orbits are (for example) *expanding*, as revealed by the apparent longitude accelerations, there is an effect on the lunar nodal rate:

$$\frac{\partial\dot{\Omega}}{\partial n_m} = +.00375; \qquad \frac{\partial\dot{\Omega}}{\partial n_s} = -.1038^{\dagger} \tag{13}$$

which yields the correction to nodal acceleration:

$$\ddot{\Omega} = \frac{\partial\dot{\Omega}}{\partial n_m}\,\dot{n}_m + \frac{\partial\dot{\Omega}}{\partial n_s}\,\dot{n}_s = .00375\dot{n}_m - .1038\dot{n}_s \tag{14}$$

If the orbits are expanding, then the nodal rate must change accordingly, since it arises from gravitational perturbations. This means that Brown (1919) has neglected the effect of the tidally induced expansion of the lunar orbit on the node. For an $\dot{n}_t$ near $-30''\mathrm{cy}^{-2}$, equations (14) and (9) yield an additive correction of about $+1.5\ ''\mathrm{cy}^{-2}$ to the apparent $\dot{n}_t$ on the standard ephemeris. This correction to $\ddot{\Omega}$ (and hence $\dot{n}_t$) was made in Muller (1975) and is therefore already reflected in the

† Misprinted +.1559 in some Deprit & Henrard preprints.

tabular computations of the eclipses below. When the final value of $\dot{n}_t$ is adopted in the lunar ephemeris, the equivalent change in $\ddot{\Omega}$ required by (14) must be entered in the coefficient G_t of nodal acceleration (2).

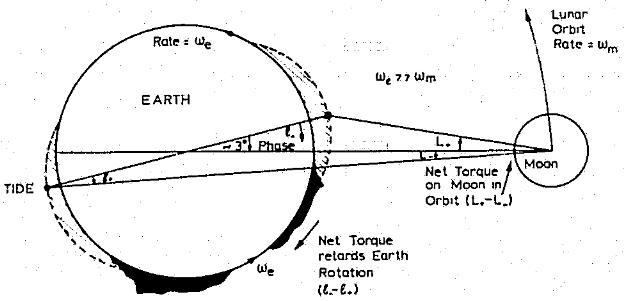
The question of a rate of change of the Gravitational Constant is subtle when it comes to effects on the lunar node. The partials in (14) from sun and moon are such that they nearly cancel when both orbits expand proportionally, and nodal effects are rather small. This question will be considered below, with suitable coefficients placed in the equations of condition.

The Earth's Rotational Acceleration

Since the earth rotates on its axis faster than the moon orbits about the earth, the nearer tidal bulge on the earth is dragged ahead of the moon. The torque on the moon arises from the net difference between the gravitational attractions exerted by the moon on the two tidal bulges, Figure 3.

Figure 3

Earth - Moon Tidal Couple



The nearer bulge is larger, and closer, and the moon is accelerated (gains energy), and moves farther from the earth, slows down, and exhibits a *negative* apparent acceleration in longitude. The earth's rotation is also negatively accelerated.

An exceptionally clear analysis of the different time scales used in Astronomy is to be found in Mulholland (1972). Ephemeris Time (ET) is the independent argument of the ephemeris required for this analysis - equations (1) through (3). Universal Time (UT) is the measure of the rotational position of the earth with respect to the sun defined by equation (3). The earth's rotational behavior is measured by examining the function ΔT defined by:

$$\Delta T = ET - UT \tag{15}$$

as a function of time (*e.g.* from -1375 to +1975), where ET and UT are the readings on the Ephemeris and Universal Time "clocks" at various instantaneous epochs of observation. *If* the ephemeris of the sun (3) is correct, then the observed values of ΔT in the presence of an acceleration in the earth's rotation $\dot{\omega}/\omega$ will satisfy:

$$\Delta T = C \cdot T^2 \text{ where } \dot{\omega}/\omega = kC \tag{16}$$

If C and $\dot{\omega}/\omega$ are in the same units, k = 2. It has been customary to express $\dot{n}$ in arcseconds per century squared, "cy^{-2}, $\dot{\omega}/\omega$ and $\dot{G}/G$ in fractional parts per year, $10^{-11}yr^{-1}$. For simplicity in this paper, the expressions for units conversions will be given without the awkward units, for example, the "k" of equation (16):

$$\dot{\omega}/\omega = -.635\,C \tag{17}$$

Previous analyses of the *ancient* observations assumed that (3) was sufficiently correct, and used (16) to define and find $\dot{\omega}/\omega$ from ΔT values at many epochs of observation throughout history; see for example Newton (1970) page 6, Fotheringham (1920), Curott (1966), Dicke (1966) and (1969), and Stephenson (1972). Studies of modern observations, including Spencer Jones (1939), Clemence (1948), Brouwer (1952) and Van der Waerden (1961) all made use of the expression defining ΔT:

$$\Delta T = A + B \cdot T + C \cdot T^2 \tag{18}$$

The A and B coefficients in (18) are equivalent to the I_o and J_o coefficients in (3), and A & B will be zero if and only if I_o & J_o are correct. Muller (1975) citing analysis by Martin (1972) shows that the data available to Newcomb (1895) were insufficient to adequately determine I_o and J_o in (3), and the substantial value of coefficient B found here, equation (62) below, and in Brouwer (1952) for example, is real and essential to finding an accurate value of $\dot{\omega}/\omega$ from the ancient observations. One could correct (3), but it is customary to leave the solar ephemeris fixed, and correct, instead, the form (18), and that is the approach used here. Muller (1975) and Muller & Stephenson (1975) show that the adoption of the simplifying assumption (16) by Newton (1970) and (1972) gave rise to his apparent observational conclusion that the accelerations had varied substantially over historical time, contrary to the findings here and in the other references. This modeling simplification also tended to substantially overestimate the true, nontidal portion of $\dot{\omega}/\omega$, $\dot{\omega}/\omega_{TNT}$ in Newton (1970), and this parameter will be considered in the next section.

Earth - Moon Tidal Interaction

Conservation of angular momentum in the earth-moon system implies that the tidally induced accelerations of the earth's rotation $\dot{\omega}/\omega_t$ and the moon's longitude $\dot{n}_t$ are related:

$$\dot{\omega}/\omega_t = 0.935\,\dot{n}_t \quad \text{(lunar tide alone)} \tag{19}$$

The solar tide must be added to the lunar contribution. Jeffreys (1962) gives equations for computing the ratio between the lunar and solar tide, and Newton (1968) evaluated them with the results: 5:1 for linear tides, 3.8:1 for quadratic (shallow sea) tides. These analyses used approximations to the tidal model, and Lambeck (1975) is the most recent work and carefully distinguishes the contributions of sun and moon. Kaula (private communication) notes that Lambeck's analysis implies:

$$\dot{\omega}/\omega_t = 1.224\,\dot{n}_t \quad \text{(lunar + solar tides)} \tag{20}$$

Transfer of angular momentum from the solar tide to the earth's orbit is negligible. There is a significant solar atmospheric tide to be accounted for, which Munk & MacDonald (1960) estimated at $+1.6 \times 10^{-11}\,\text{yr}^{-1}$. Newton (1970) used $+2.7$, but Lambeck (1975) finds from Kertz *et.al.* (1970):

$$\delta\dot{\omega}/\omega_t = +\,1.1 \times 10^{-11}\,\text{yr}^{-1} \tag{21}$$

Lambeck notes the error in Newton, and adoption of (21) is undoubtedly the best available result. To compute the *apparent* nontidal acceleration from the observed total acceleration:

$$\dot{\omega}/\omega_{ANT} = \dot{\omega}/\omega - 1.224\,\dot{n}_t - 1.1 \tag{22}$$

where the units are those adopted above.

Observational Determination of the Accelerations

To extract the several highly desirable astronomical and geophysical parameters from the observations, it is necessary to *independently* determine (*i.e.* separate) the parameters $\dot{\omega}/\omega$ and $\dot{n}$. As Newton (1970), page 3, and others have noted, the determination of one parameter given the other is done to a high precision, whereas the solution as independent parameters is difficult and the sensitivity is not all that one might desire (even two parameter systems can be highly correlated!). The solutions given in this paper, Muller (1975), Muller & Stephenson (1975), and Newton (1970) page 272 all agree in the parameter ΔT (at a *given* $\dot{n}$) within about 200 seconds of time (see figure 13 below). For $\dot{\omega}/\omega$ as an *independent* parameter, the typical uncertainty for an ancient epoch (near 0) will be at least an order of magnitude greater in absolute ΔT. The high degree of correlation between $\dot{n}$ and $\dot{\omega}/\omega$ can be seen in figure 2. Consider the effect of a change in $\dot{n}$ of $5''\text{cy}^{-2}$ at the epoch -100 ($T = -20$cy). This changes the computed lunar longitude $\Delta\lambda$ at the *same* ET by about 1000". As we saw above, this can be *cancelled* by an appropriate change in ET − UT, leaving only the small change in latitude at conjunction $\Delta\phi \approx 8''$ from (6). In a solar eclipse path on earth, this 8" is equivalent to about 15km in latitude change, which for an inclination of 30° between the eclipse path limit and a line of equal latitude gives about 30km of longitude change with which to measure the independent ΔT change between the parameters. Total eclipse paths are typically 100 to 200km wide, so it is apparent that a few sound observations can probably resolve $\dot{n}$ to $5\,''\text{cy}^{-2}$ or better.

In timed lunar observations, the situation is much worse. The earth will rotate the above 30km in about 1.5 minutes, while the moon will cover the roughly 80" of residual longitude change at conjunction (C_0 to $M_{\dot{n}}$ in figure 2) in about 2.5 minutes. The *net* effect of these timing variations in a given timed observation will be some linear combination of these numbers with coefficients of less than unity applied to each, depending upon the circumstances of the observation, or something in the range of 1 to 3 minutes typically. I know of only one observation from the ancient world which even approaches this level of accuracy: the Babylonian timing of the beginning of a partial eclipse of the sun on 26 September −321 as reported by Fotheringham (1935). It turns out to have a potential observational accuracy of about 5 minutes of time, and a sensitivity of about 1 minute of time per $4\,''\text{cy}^{-2}$ in $\dot{n}$ ($\partial\Delta T/\partial\dot{n}$ in figure 9). It can therefore bound $\dot{n}$ within roughly $\pm 20\,''\text{cy}^{-2}$, and it is by far the most precise timed observation we have from that era! Muller (1975) goes on to justify the conclusion that ancient timed lunar observations and untimed lunar eclipse data have insufficient sensitivity to have any material effect on the parameter solutions. These data are therefore not considered further.

The timed *solar* observations (equinoxes) are quite another matter since these data are independent of the moon and measure ΔT directly. They therefore separate the parameters with the full accuracy of the observations themselves, and a precision of an hour can be very useful! These data are considered below, in Muller (1975) and Newton (1970).

The *mean* linear relationship between the parameters can be computed from the mean motions of the moon and earth's rotation, and is a useful formula to note. For observations of the moon with respect to the sun:

$$\frac{\partial\dot{\omega}/\omega}{\partial\dot{n}} = +.622; \quad \dot{\omega}/\omega - .622\,\dot{n} = L \tag{23}$$

where L is the observed (solved-for) constant term in the linear relationship between the parameters. Individual observations will have partial derivatives which can differ from .622 by up to about 10%, the limit being imposed by the factor Tan I in (6). This is another way of viewing the separation of the parameters.

Stephenson (1972), and Muller & Stephenson (1975) argued convincingly that many partial eclipse data were subject to very severe systematic bias due to population distributions. To be safe from this *avoidable* bias one must use *total* eclipses only. Muller (1975) also showed that the partial eclipse data had insignificant sensitivity in separating the parameters, and could safely be deleted.

Minimum Deletion Linear Inequalities Filter

An important finding of Muller (1975) was that least squares is not the optimum statistical filter in the reduction of total solar eclipse data. Every study of ancient eclipses, except Fotheringham (1920), converted each eclipse observation into a weighted equation of condition suitable for least squares processing. This will work reasonably well if the desired (or achievable) final solution uncertainties are large compared with typical path *widths*. Stephenson (1972) who relied heavily on the most ancient data in his mean solutions, and Curott (1966) who sought limited accuracy, lost comparatively little by this statistical simplification. There are two *unnecessary* losses in sensitivity which result from using least squares equations of condition with total solar eclipses. [1] The hard limits (see below) of the path edge are approximated by the Gaussian distribution curve and lose sharpness. [2] The estimator tends to force solutions towards the *center* of paths, when stations of observation within the path are equally likely. Muller (1975) found Newton's (1970) solutions to be capable of considerable variation: "It has been possible with Newton's data to obtain reasonably stable solutions for $\dot{n}$ as an independent parameter in the range -29 to -46 "cy^{-2}." Newton (1970) and (1972) largely dissipated his accuracy through this approach because he placed heavy demands on the solutions including higher precision and an attempt to see if the accelerations varied (necessitating solutions at several epochs). Muller (1975) and this paper use the path *limits* (not just widths) as bounds on the possible variation of the parameters. Such a situation is unusual in observational science, but not unknown, and the exact mathematical approach to the statistical reduction of such data is to use systems of linear inequalities.

Figure 4

Observational Equations of Condition

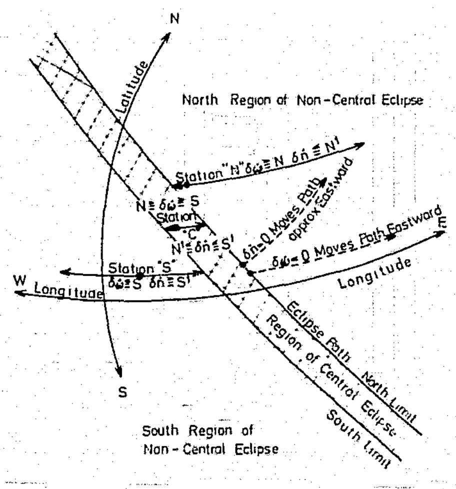


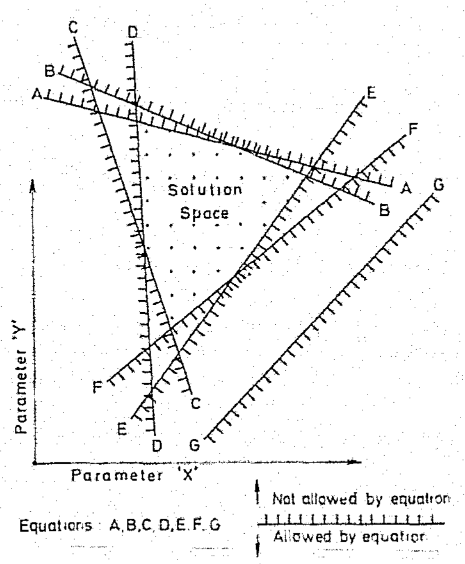
Figure 4 illustrates the parameterization of a total eclipse observation. The eclipse path is calculated for the nominal parameter values. Changing $\dot{\omega}/\omega$ moves the computed path directly east-west, whereas changing $\dot{n}$ moves the path in a direction which is generally not east-west, differing by the $\Delta\phi$ at conjunction noted in figure 2. If an observation places station "C" within the path of a known eclipse, both upper and lower limits are placed on the maximum range of the two parameters. If N and S are the values for the change in the parameter $\delta\dot{\omega}$ which brings the observation station onto the north and south limits respectively, N^1 and S^1 for $\delta\dot{n}$, then figure 4 gives the inequality equations of condition on the parameters. There are corresponding equations for an observation requiring the station to be outside the path (strict denial of totality). Each parameter is bounded by two equations of condition, which yield a pair of linear *boundaries* corresponding to each equation in the solution space for the two parameters, which in principle can be plotted as in Figure 5.

Each equation of condition implies that values of the parameters on one side of the line (cross-hatched) are ruled out. The logical intersection of these constraint equations is defined as the solution space, within which the true solution must lie (assuming correct observations). Figure 5 is similar to the presentation of Fotheringham (1920), although he did not use an orthogonal pair of parameters.

A given set of linear inequalities will either provide a solution space with finite area, or the set of equations will be inconsistent and the space will be empty. The latter would hold if equation G in figure 5 was replaced by its logical inverse ~G (reversing the inequality). A central characteristic

Figure 5

Schematic of Solution Space



(and strength) of linear inequalities filtering is that a given equation is either correct, or incorrect (an eclipse is either correctly assigned or not), and if correct we will see below that the limits are very sharp indeed compared to the path width. An erroneous equation may (but need not necessarily) create an inconsistent set with an empty solution space.

The minimum deletion linear inequalities filter is defined as follows. Each equation of condition will have a truth probability assigned to it on the basis of historical soundness and observational characteristics in the manner of Muller & Stephenson (1975), the actual values used here being from Muller (1975). If the actual set of linear inequalities provides a solution space, it will be used subject to the data consistency test below. If the set is inconsistent, the group of observations with the minimum joint probability of truth, whose deletion creates a solution space, will be thrown out. This is exactly analogous to the usual process of deleting a few bad data in a least squares fit, when their residuals after the fit exceed a given multiple of the overall standard deviation of the fit (often $3 \cdot \sigma$).

No solution by least squares or any other filter is any better than that which is shown to be sound by an adequate data consistency check (redundancy). Erroneous equations will have one of three effects on the linear inequalities filter. [a] The equation is distant from the bounded solution space, but is *consistent* with it (for example equation G in figure 5). The result: no effect on the solution or quoted uncertainties! [b] The equation is distant from the bounded solution space, but is inconsistent (as would be the case if we replaced G with ∿G). In this case, it will be discovered and deleted in the minimum deletion set of inconsistent equations, providing only that good data outnumbers the bad by an adequate margin (without which we have failed in any case). The result: no effect *whatever* on the solution! [c] The erroneous equation intersects the true solution space, thereby reducing its area. There are two possibilities. {i} The offending equation forms a bound near another (presumably correct) equation which limits the solution space. If, for example, equation E is wrong, there is no serious effect on the solution space, and the quoted result (centroid of the space) and error estimates (distances to the bounds from the centroid) are hardly affected. Equation E would not be identified as "wrong" but it would have a negligible (or at least very limited) effect on the solution space. {ii} The worst possibility is illustrated by replacing equation D with ∿D. Now the "solution space" will be incorrectly identified as the small region between equations A, C, and ∿D. The (centroid) estimate of the parameters will be moved by about 1.5 standard deviations, which is serious enough to avoid if possible, and the solution uncertainty would be grossly underestimated. This can be avoided by demanding that a simple data consistency test be satisfied. We will simply delete any single equation (correct or not) whose removal would *substantially* increase the solution space size (as removing ∿D surely would). If ∿D was incorrect, we provide a solution space very close to the correct one. If the correct space was the small one, we have provided a more conservative solution consistent with the estimated errors. Wrongly identified large solar eclipse observations have a tendency to be *grossly* in error (*e.g.* assigning Rome instead of Athens as the place of observation). Bad equations do not have a high probability of intersecting the small solution space, class [c] above, and even if one does do so, application of the data consistency (redundancy) test will minimize its effect on the quoted solution. A limited number of erroneous equations have a high probability of having no effect *at all* on the solution!

We have seen two extremely important advantages of this filter for the processing of historical eclipse data. First, the more in error an erroneous equation is, the *less* the chance that it can have any effect on the solution. Second, a chance erroneous equation intersecting the solution space can have at worst only a modest influence because of the adopted data consistency requirement. This is precisely the kind of stability we need, because these data are either correct and very precise in the imposed limits, or they are very, very wrong. This stability is in marked contrast to least squares, where really bad equations of condition are disastrously emphasized by the least *squares* rule. It is worth repeating that least squares equations of condition are only a poor approximation to the actual linear inequalities imposed by central eclipse observations, and losing sensitivity by using least squares is quite unnecessary (and very unwise in view of the relative paucity of good data).

Muller & Stephenson (1975) undertook to evaluate the equations of condition by a rather complex iterative scheme. This was simplified in Muller (1975) to a process of single parameter solutions using partials with respect to the observables as conducted below. Determination of the solutions is undertaken directly in ΔT, and since we have reasonable a-priori on the final solutions, it is convenient to express the observational bounds in $\delta\Delta T$:

$$\delta\Delta T = \Delta T_{observed} - \Delta T_{nominal} \tag{24}$$

Partial derivatives of ΔT with respect to $\dot{n}$ can be determined by numerical calculation, thus allowing this parameter to be determined, while $\dot{\omega}/\omega$ will follow from the fit of equation (18) to the bounds in ΔT versus time, and use of (17).

Observation of Large Solar Eclipses

A total eclipse of the sun begins quietly, a small nick appearing on the western limb of the sun. Those unaware of the impending eclipse notice nothing, unless clouds or a chance reflection in a pool of water happens to afford the right degree of solar filtering. The moon moves inexorably across the sun's disk, taking between 60 and 90 minutes in the crossing. At 90% eclipse, the darkness is equivalent to moderate clouds and the curious casual observer might glance up to see what had obscured the sun in a cloudless sky - but he would be quite dazzled by the sun and would not detect the cause. A few minutes later, at 99% eclipse, the air would be chilling perceptibly, and the darkening would equal that of a very heavy overcast. If our unsuspecting observer now glances at the sun, he is just able to perceive the brilliant solar crescent. Something is decidedly wrong. It is barely 60 seconds to sunset, at high noon!

Day birds hurry to roost, night birds fly out. Animals and even insects hush without exception, every living creature tensely aware that *something* is about to happen. Suddenly, the sun is cut to a thin line, and shadows begin dancing on the ground as at the bottom of a wind-blown pool of sunlit water. All eyes turn in terror or disbelief from the ground, drawn to the sun. The thin line of crescent breaks up into incandescent pulsing beads of intense light, far brighter than Venus *stars*; the beads seem to move, to corpusculate, flashing on and off. During the previous hour, the sun has dimmed a thousand times. In these last four seconds, it falls *another* thousandfold, and the umbral shadow races in from the west at three times the speed of sound. The sun *instantly* disappears completely, only a thin ring of pearly light visible about the moon's black rim, punctuated by tiny, blood-red licking flames! The priest falls to his knees murmuring the *miseri-mei*; screams of terror die unborn in the dry throats of thousands; the only sound is the flapping wings of disoriented day-birds flopping on the ground unable to see to fly, and night birds leaving for the night harvest of insects -

The sky to the west is brooding black and darkening, while to the other compass points, the red, thin clouds of distant sunset reflect saffron light garishly into the intense faces staring hypnotized at the departed sun. The air chills, dew falls, a light breeze springs up from nowhere. The night darkens, and stars appear, a few at first, many more as the eyes adjust to the darkness. The sun stands now in a halo of pearly, flamelike streamers which seem to *grow* as the endless time passes - extending first to five, then ten solar diameters like a great celestial black-centered, filamentary-petaled flower. The deep sunset sky now extends the full 360° of the horizon in all directions. All minds are obscessed with the one thought, the unspeakable question: will God return the sun to us, or is this His final vengeance?

All eyes are raised to the dethroned sun. Instantly, the night is turned to day as the first bead of light escapes the moon's limb. To dark adapted eyes, it irradiates incandescently, throwing a bolt of blinding light in the pattern of a great cone of streamers arching down from the heavens directly into each bared soul, personally. For a brief moment, it takes the aspect of a great ring set with a diamond of unearthly brilliance, bound around the black lunar disk by the thin line of rapidly fading inner corona; then it is too painful to look any more. The ground ripples again with the fantastic shadow shapes, and the western sky brightens rapidly, even as the shadow visibly retreats into the still darkened eastern horizon. Maybe - just maybe - it *isn't* the End of the World after all . . .

If you haven't thrown this paper in the general direction of the circular file, you are probably thinking that this description is either grossly exaggerated, or is a misguided attempt at literary prose. The latter might be true, but the former is not. It is difficult to convey the impressive spectacle afforded by a total solar eclipse with words; it is really impossible to do so with photographs - though Muller (1975) references some plates from the literature which provide a rare hint of the impression.

I have witnessed four total eclipses of the sun, and certainly cannot claim to have witnessed every wonder of nature, but there is little room for doubt that a total solar eclipse is the most humbling and impressive natural spectacle which nature affords us on this planet. The twentieth century astronomer, and aboriginal native alike, share the same feelings - an impression of mind and soul which bridges the ages of man as perhaps no other event can do. We are dealing in this study with a basically subjective phenomenon, and it is important to understand the subjective impression. The awesome impact of such an eclipse is undoubtedly the reason why we find so many more historical references than we have any right to expect. If an eclipse happens in a center of civilized man, it is recorded, a hundredfold, and this explains why we find such a deep historically recorded tradition in this area.

Scientists correctly prefer quantitative proofs when it comes to matters of observational resolution and accuracy. There have been two substantial experiments which show that *untrained* observers can reliably distinguish the difference between a total and nontotal eclipse with a resolution at the path edge of the order 200 meters! The eclipse data recorded and analyzed by Halley (1715) is marvelously detailed, and as reduced in Muller (1975), achieves this level of resolution. Halley even correctly notes the conditions of the Baily beads in this eclipse. The *Transactions of the Illuminating Engineering Society* (1925) documents an experiment in New York City which shows that a resolution of 200 meters is achievable in practice.

Figure 6

Schematic Total Eclipse Light Curve

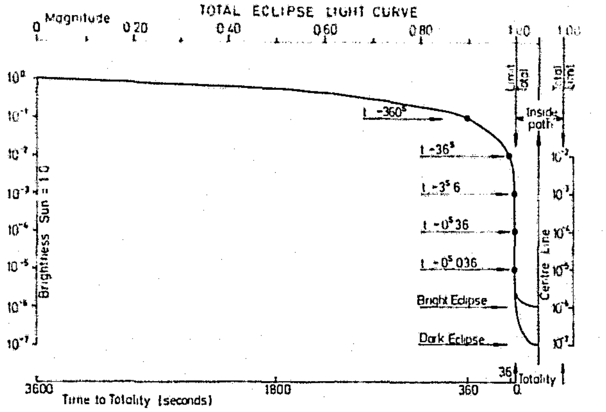


Figure 7

Typical Observational Details

Strong Indications of Totality

1 The sun disappeared
2 The sun *suddenly* disappeared
3 Darkness *suddenly* fell
4 No part of the sun remained in view
5 Sun completely covered
6 Ring or *bag* seen around the sun
7 The sun *set* in the daytime
8 Chinese astronomer's term *chi* (complete)

Ambiguous Indicators

9 Stars seen
10 Great eclipse
11 Eclipse like none before it
12 Great darkness
13 Sign in the sun
14 *Horrible* eclipse

Strong Indications of Nontotality

15 Crescent *rotated* around the sun
16 Flashing, corpusculating lights
17 Sun appeared like a (two to five) day moon

Figure 6 is a schematic of the light curve in a total eclipse, and is similar to the observed curve provided in the *Transactions of the Illuminating Engineering Society* (1925). The key to the high geometric resolution at the path edge is the very bright light provided by the smallest portion of uneclipsed sun. The corona (solar atmosphere) visible in an eclipse has a brightness of about 10^{-7} of the whole solar disk. Light scattered into the eclipse zone from outside it can increase this by about a factor of ten. In the first few km outside the path limit, the light level rises to 10^{-3} of the sun, or three to four orders of magnitude. We do not need resolution of 200m for the purposes of this study of ancient solar eclipses, and the km level of resolution guaranteed by the very steep lighting curve is more than adequate. In a typical annular eclipse, several percent of the sun remains uneclipsed. For this reason, very few annular eclipses have been adequately observed in antiquity. It is important to note that until the eclipse reaches 99%, the solar crescent is not discernable by the naked eye in a clear sky. The resolution of naked eye observation of a total solar eclipse is of the same order as modern, manual telescopic observations of the moon. That is why the ancient eclipse observations are so powerful, and it is crucial to use the path limits rather than the widths in the data reduction.

In the reliable interpretation of historical records, one must be conversant with most of the subjective phenomena in a total eclipse. Newton (1970) and (1972) pioneered in this area, Stephenson (1972) and Muller (1975) carrying on. All agree that the statement "stars seen" is not a reliable indication of totality. Figure 7 lists the observational indicators adopted in Muller (1975). Following Stephenson (1972), Muller & Stephenson (1975), Muller (1975) and this paper admit an observation to the final solution only if it unambiguously indicates the *precise* magnitude achieved (total or denial of totality). A statement of very large magnitude is acceptable for *very* ancient data (*ca.* -1000).

For an eclipse record to be usable in the astronomical determinations, three criteria must be satisfied with a sufficiently high probability of truth (greater than .50 in this study). They are listed in Figure 8. Identification (of which eclipse was observed) is not usually the cause of fatal difficulties, although there are such notable exceptions as the Eclipse of Hipparchus. Total eclipses at a given station are very rare (once per 360 years on average), and it is usually enough to know the King's reign, or a rough date, in order to unambiguously identify the eclipse. It is *not* necessary to have the time of day, the day, or even the year, and the great strength of this data is that it is untimed. The question of magnitude has already been discussed, and the critical detail which is almost invariably omitted is the observer's *location*. He no doubt feels it is implied, or the need (to twentieth century science) for him to state it simply fails to occur to him! To appreciate this difficulty it is only necessary to read a few hundred pages from Newton (1970), (1972), or Stephenson (1972)!

Figure 8

Three Observational Requirements

I Identification of *which* eclipse was observed

II The *precise* magnitude observed

III The place of observation (usually from context)

The Eclipse Observations

The historical analysis underlying the selection of the data adopted in this study comes almost entirely from Newton (1970), (1972), and Stephenson (1972). The sources include original records from the ancient near east including the Babylonian Astronomical Texts and tablets from early civilizations in this area; the Greek and Roman classics, upon which every study before Curott (1966) relied exclusively, but which are found to be a small source in this paper and Stephenson (1972); the Chinese Annals and Astronomical Treatises which Stephenson pioneered, thereby substantially increasing the available reliable data; and the Medieval Chronicles and Texts considered by Newton (1972) and Stephenson (1972). Most of the observations noted in this paper are analyzed in Muller & Stephenson (1975) according to the criteria we adopted in our collaboration. Muller (1975) contains the final analysis of all observations.

Figure 9 lists the observations admitted in Muller (1975) with the columns as follows: the date; place of observation with its latitude and longitude 0 to 360° east of Greenwich; observed phase (Total, Annular, Partial {totality denied}, or timed Contact); lower and upper limits of bounds on $\delta\Delta T$ as defined in (24), where the nominal ΔT is from line one of Figure 10 and where the coefficients A, B and C are entered in (18); the partials of ΔT with respect to $\dot{n}$ and $\ddot{\Omega}$ in the units of this paper; and the probability that the observation is valid as assigned in Muller (1975). One of the data in Muller (1975) has since been ruled out, thanks to valuable correspondence from G. J. Toomer, who noted that Bagdad was not yet founded as a city in 693 AD! As can be seen below, data at this epoch plays no role in defining the mean solutions. It is humorous in retrospect that we checked the founding dates for these cities carefully for every data point *except* this one which was inadvertently overlooked. This is a perfect example of the law of perversity: the worst that can happen, will happen, if you don't prevent it!

Toomer (1974) provides a fascinating argument which identifies the Eclipse of Hipparchus as that of -189 March 14 on the basis that only the elements of this eclipse give the lunar parallax which Hipparchus quotes as resulting from his eclipse reduction! Unfortunately it was necessary to make some assumptions about how Hipparchus would have simplified his calculation, and some doubt can still remain about this most intriguing of all eclipse identifications. Even if this date is taken, there is considerable uncertainty as to what part of the Hellespont *region* provided the observation. It is likely that this data will never be sufficiently defined to be of use in solving for the accelerations.

Figure 9

Listing of Observations

Date	Place	Lat	Lon	Φ	LL	UL	$\frac{\partial\Delta T}{\partial\dot{n}}$	$\frac{\partial\Delta T}{\partial\ddot{\Omega}}$	P(t)
1567 04 09	Rome	41.90	12.48	T	+110	+160	1.4	+ 22	1.00
1560 08 21	Coimbra	40.20	351.58	T	--	+230	- 1	- 10	1.00
1361 05 05	Mt Sumelas	40.67	39.64	T	--	+ 95	0	0	.90
1267 05 25	Constant.	41.02	28.98	T	--	+150	+ 4	+ 50	.80
1241 10 06	Stade	53.60	9.48	T	-160	--	- 2	- 30	1.00
1241 10 06	Lambach	48.10	13.90	T	-265	--	- 2	- 30	.80
1241 10 06	Sheftlarn	47.92	11.42	T	--	+ 42	- 2	- 30	.80
1239 06 03	Toledo	39.87	355.97	T	+ 10	--	- 1	- 12	.95
1239 06 03	Cerrato	41.95	355.48	T	--	+276	- 1	- 12	.98
1221 05 23	Kerulen R.	48.18	115.90	T	--	+214	- 4	- 50	.98
1178 09 13	Vigeois	45.38	1.52	P	--	+185	+ 2	+ 24	.98
1133 08 02	Salzburg	47.80	13.06	T	--	+ 70	- 2	- 24	1.00
1133 08 02	Vysehrad	50.06	14.42	P	--	+ 75	- 2	- 24	1.00
1133 08 02	Kloster.[†]	50.87	6.07	T	--	- 82	- 2	- 24	.80
1133 08 02	Corvei	51.82	9.43	T	-180	--	- 2	- 24	.80
1133 08 02	Liège	50.63	5.58	P	-315	--	- 2	- 24	.70
1124 08 11	Novgorod	58.50	31.33	T	-156	--	+ 5	+ 56	1.00
1079 07 01	Alcobaca	38.73	350.87	T	--	+ 5	- 3	- 36	.80
968 12 22	Constant.	41.02	28.98	T	-470	--	-14	-128	1.00
916 06 17	Cordoba	37.88	355.23	T	--	+280	- 5	- 64	.80
522 06 10	Nanking	32.03	118.78	T	--	+100	+ 7	+ 87	.50
516 04 18	Nanking	32.03	118.78	A	--	+130	-13	-128	.50
484 01 14	Athens	37.98	23.73	T	--	+154	+12	+157	.50
120 01 18	Lo-yang	34.70	112.47	P	+ 90	--	+13	+185	.98
71 03 20	Chaeronia	38.40	22.90	T	- 62	+ 38	-11	-110	.50
65 12 16	Kuang-ling	32.42	119.45	T	--	+ 65	-15	-150	.80
-135 04 15	Babylon	32.55	44.42	T	--	+125	-16	-170	1.00
-180 03 04	Ch'ang-an	34.34	108.90	T	--	+146	+17	+222	.95
-187 07 17	Rome	41.90	12.50	T	- 53	+247	+19	+267	.50
-197 08 07	Ch'ang-an	34.34	108.90	A	--	+245	-54	-540	.90
-321 09 26	Babylon[§]	32.55	44.42	C	-450	-210	-15	- 73	1.00
-600 09 20	Ying	30.34	112.25	T	-100	+760	-26	-262	.80
-708 07 17	Chü-fu	35.53	117.02	T	-350	+575	-23	-244	.90
-762 06 15	Nineveh[¶]	36.40	43.13	T	+216	--	+50	+650	.30
-1130 09 30	Gibeon	31.85	35.20	T	-720	+160	+26	+365	.50
-1329 06 14	An-yang	36.07	114.33	T	-515	- 40	-35	-390	.80
-1374 05 05	Ugarit	35.62	35.78	T	+130	1125	100	1340	.80

[†]Klosterrath: only inconsistent observation in the set.
[§]Timed contact: error is larger than expected, see text.
[¶]Was not included in solution: perhaps should have been.

Figure 10

Nominal Parameter Starting Conditions

#	Run	$\dot{n}$	$\dot{\omega}/\omega$	$\ddot{\Omega}$	A	B	C
1	Nominal	-32.5	-26.0	0.000	25.	121.	40.95
2	$\partial\Delta T/\partial\dot{n}$	-37.5	-29.1	0.000	25.	121.	45.82
3	$\partial\Delta T/\partial\ddot{\Omega}$	-32.5	-26.0	-.412	25.	121.	40.95
	Units	$''\mathrm{cy}^{-2}$	$10^{-11}\mathrm{yr}^{-1}$	$''\mathrm{cy}^{-2}$	s	$\mathrm{s\cdot cy}^{-1}$	$\frac{1}{2}\mathrm{s\cdot cy}^{-2}$

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Two observations in figure 9 are controversial, and it seems that every worker in this field manages to choose one or two observations which nobody else will touch! For -1130 September 30 we have supporting analysis, Sawyer (1972) and Stephenson (1975), but several recent correspondents have noted evidence that Joshua may be dated in the 15th century BC. If this is the case, then Stephenson (1975) would prove that there is no real eclipse associated with Joshua and the observation would have to be dropped. Some new arguments are given in Muller (1975) to support Plutarch's famous eclipse of 71 March 20. Toomer (private communication) points out that Plutarch's reference to the corona occurs in the very same words in Cleomedes in a context drawn from earlier Greek astronomers. This shakes my confidence in the argument of Muller (1975) because the key point there is that Plutarch is *behaving* like an eye witness. It is still possible that he would use a particularly apt phrase from an earlier source, but the argument is weakened. It will be seen below that these two data can be deleted without materially changing the solutions, and the remaining observation set appears very strong.

The Equinox Observations

An observation of a solar equinox (declination zero) given in local time for a known place provides a direct measure of ΔT (and hence $\dot{\omega}/\omega$) independent of the moon. As shown in Newton (1970), a small bias (by ancient standards) in the setting of an equator plane is fatal to these data unless it can be independently determined. As Newton notes, this error has the opposite effect on ΔT determined from vernal and autumnal equinox observations, allowing a solution for both equator bias and ΔT if data from both seasons is available from the same instrument. The estimate of ΔT from equinoxes is merely the difference between the observed Greenwich Time (UT) of the event, and the ET at which the computed position of the sun equals the observed position:

$$\Delta T_e = ET_{sun} - UT_{obs} \qquad (25)$$

To estimate ΔT in the presence of an unknown bias in the instrument equator, given sufficient data to define both vernal and autumnal values of ΔT:

$$\Delta T = \tfrac{1}{2}(\Delta T_v - \Delta T_a) \qquad (26)$$

and the equator declination bias follows from:

$$\Delta\delta = (\Delta T_{obs} - \Delta T_v)\cdot\dot{\delta} \qquad (27)$$

Newton (1970) argues that the solstices have insufficient sensitivity to be useful, and they are not considered further in this paper.

The equinoxes of Hipparchus are quoted in Ptolemy's *Almagest*. Fotheringham (1918) is the first secondary source analysis. Newton (1970) and Muller (1975) analyze these data, and join with Fotheringham (1918) in noting the peculiar problems associated with Hipparchus' rounding the observations to even quarter days. The Tropical year is less than the Julian year by only 0.188 hours, and this means that if *perfect* observations are rounded to 6 hours, there is a series of about 32 observations at exact intervals of 6 hours, and then a discontinuity of 6 hours occurs as the annual deficiency of 0.188 hours finally reaches the rounding level. This curious phenomenon makes the interpretation of Hipparchus' equinoxes a matter of finding the *crossover* point in a sequence of observations, inferring the effective ΔT implied by the entire series. Newton (1970) notes that it is not correct to merely average the observations when the rounding tolerance is large compared with the observational accuracy. Muller (1975) analyzes these data in much the same way as Newton (1970) except: [1] the unpublished solar tables used by Newton are replaced by evaluation of the solar theory; [2] a more precise computation of the separate autumnal and vernal sequences is made; [3] the approximate observational accuracy is *estimated* from the observations themselves to replace Newton's heuristic estimate of 1 hour; [4] the observation made on the κρικος at Alexandria reported in the *Almagest* is included in the error analysis and solution estimate. The final solution is identical to Newton (1970) except for the slightly reduced estimated error:

$$\Delta T_H = +\,4.20 \pm 0.8 \text{ hours}; \quad \textit{epoch } -145 \qquad (28)$$

To obtain $\dot{\omega}/\omega$ it is necessary to employ equations (17) and (18). An approximate version of (18) would be sufficient, but we might as well look ahead and use the best available result (from the ancient eclipses below), equations (42) through (48), and using the last:

$$\dot{\omega}/\omega = -26.5 \pm 4.4 \times 10^{-11}\,\text{yr}^{-1}; \quad \textit{epoch } -145 \qquad (29)$$

The Islamic Equinoxes are analyzed by Newton (1970) and Muller (1975), the latter differing by: [1] using an exact calculation of the solar theory instead of unpublished tables (which here makes a noticeable difference in the JED column of Figure 11 compared with Newton); [2] assigning about one-half the estimated error compared with Newton. Figure 11 indicates the date; place; longitude east of Greenwich, from which the JUD, Julian Universal Date (UT) of the observation is computed; the JED (ET) of zero solar declination from the solar theory; and the difference ΔT (JED - JUD) expressed in hours. Muller (1975) argues that observation #1 differs from noon by exactly the equation of time and was probably calculated by an editor, that the solstice (#6) is likely to be weak, and that #9 is quoted as noon exactly but has the largest residual and is therefore suspect.

Figure 12 exhibits several defensible choices

Figure 11

The Islamic Equinox Observations

#	Yr Mo Dy	Hr	Place	Lon°	JUD†	JED†	ΔT hr
1	830Sep19	12.13	Damascus§	36.3	24476.8983	6.8961	-0.053
2	830Sep19	13.00	Bagdad	44.4	24476.9129	6.8961	-0.403
3	831Mar17	2.00	Bagdad	44.4	24655.4654	5.5525	+2.090
4	831Sep19	19.00	Bagdad	44.4	24842.1629	2.1351	-0.667
5	832Mar16	8.00	Bagdad	44.4	25020.7154	0.7909	+1.812
6	832Jun18	00.00	Bagdad§	44.4	25114.3767	4.3435	-0.797
7	832Sep18	23.30	Damascus	36.3	25207.3646	7.3878	+0.557
8	844Sep18	21.40	Bagdad	44.4	29590.2629	0.2882	+0.607
9	851Sep19	12.00	Nisabour§	58.8	32146.8313	6.9797	+3.554
10	882Sep19	1.15	ar-Raqqah	39.0	43469.4342	9.4911	+1.366

†JUD - 2,000,000 days; JED is the same day as JUD.
§Questionable data, see text.

Figure 12

Islamic Equinox Means

#	Solution	ΔT hr	Δδ hr	Comment
1	Mean of all: = wts	+.806		Newton's Preference
2	All: #6 wt = ¼	+.937		Newton's Preference
3	Delete: 1,6,9	+.766		Muller's Preference
4	Bagdad set	+.708	-1.243	Removes Equator Bias
5	#8 Alone	+.607		Instrument Reset?
6	Mean of 7,8,10	+.843		Late Observations
	Adopted Mean:	+.778 ± .140		

for averaging subsets of the data to find ΔT, including the choices of Newton (1970) and Muller (1975). Newton also comments that a value near 0.7 hours could be taken by any reader as reasonable. The adopted mean and standard deviation adequately encompass the alternate data choices, and lie close to the preference of Muller (1975) on line 3. The better data are given to 1 hour, which would imply a standard deviation from roundoff of about 0.33 hr. It is likely that 4 or 5 of the observations reach this level of observational accuracy, and the adopted result is:

$$\Delta T_I = +\ 0.778 \pm 0.14 \text{ hours}; \ \textit{epoch}\ 840 \tag{30}$$

Converting to $\dot{\omega}/\omega$ as with (29) yields:

$$\dot{\omega}/\omega_I = -\ 22.2 \pm 2.8 \times 10^{-11}\,\text{yr}^{-1}; \ \textit{epoch}\ 840 \tag{31}$$

Newton (1970) provides an analysis of the stated mean positions of sun and moon given by the so-called Hakemite Tables. Muller (1975) also analyzes this presumably ephemeris position:

$$\lambda_s = 254.7658°; \ \lambda_m = 270.6868°; \ \lambda_m - \lambda_s = 15.9210° \tag{32}$$

Epoch: Noon, Cairo Mean Time 30 November 1000AD.

Newton (1970) rather arbitrarily, in the opinion of Muller (1975), assigned an uncertainty of 30 arcseconds to this "observation". This uncertainty might not be too optimistic for the observational mean; but mean with respect to what? The steps necessary to obtain a true mean ephemeris position on the *abstract* equator are subtle even for modern astronomy students.

Did the framers of this ephemeris *first* make a long series of observations and then *set* the instrument equator *before* making further ephemeris source observations? Did they write their ephemeris with respect to the best observed value of the *true* equator, or were they satisfied with an *instrument* ephemeris? Did they recognize the difference? We have seen evidence from the Islamic Equinoxes, figure 11, that the Bagdad equator was reset with improved accuracy. It is therefore probable that the Cairo instrument would also be reset at various times before, during, or after the observations were made which yielded the stated ephemeris position. We could perhaps rely on their equator if we knew that they solved for the abstract, ephemeris equator from the data, but I am unwilling to assume that they did this. It seems generous to assume that the Cairo instrument and/or ephemeris equator was three times as accurate as the computed equator bias at Bagdad near epoch 832 (line 4 of figure 12). This yields the estimate of error for the ΔT implied by the Hakemite Tables as computed by Muller (1975):

$$\Delta T_T = 3223 \pm 1500 \text{ s}; \ \textit{epoch}\ 1000 \tag{33}$$

and converting to $\dot{\omega}/\omega$ as in (29) yields:

$$\dot{\omega}/\omega_T = -\ 32.2 \pm 13 \times 10^{-11}\,\text{yr}^{-1}; \ \textit{epoch}\ 1000 \tag{34}$$

The weighted mean of equations (29), (31) and (33) provides the best estimate of $\dot{\omega}/\omega$ independent of the moon from the Timed Solar Data:

$$\dot{\omega}/\omega_{TSD} = -\ 23.8 \pm 2.3 \times 10^{-11}\,\text{yr}^{-1} \tag{35}$$

Since, as noted above, there is a very tight linear relationship between the parameters $\dot{n}$ and $\dot{\omega}/\omega$, we can compute the former value corresponding to the latter in (35). Again, looking ahead for the most precise equation (47) we find:

$$\dot{n}_{TSD} = -\ 29.1 \pm 3.7 \ \text{"cy}^{-2} \ (\text{from } \dot{\omega}/\omega_{TSD}) \tag{36}$$

Solutions from the Large Solar Eclipses

The observations accepted for inclusion in the statistical regressions have been tabulated in figure 9. Determination of the solutions for $\dot{n}$ and $\dot{\omega}/\omega$ will be undertaken in the single parameter ΔT by using the partials given in figure 9 by way of δΔT as defined in (24) where the nominals come from line 1 of figure 10. The δΔT upper and lower limits for any other parameter or coefficient choices can be

found from:

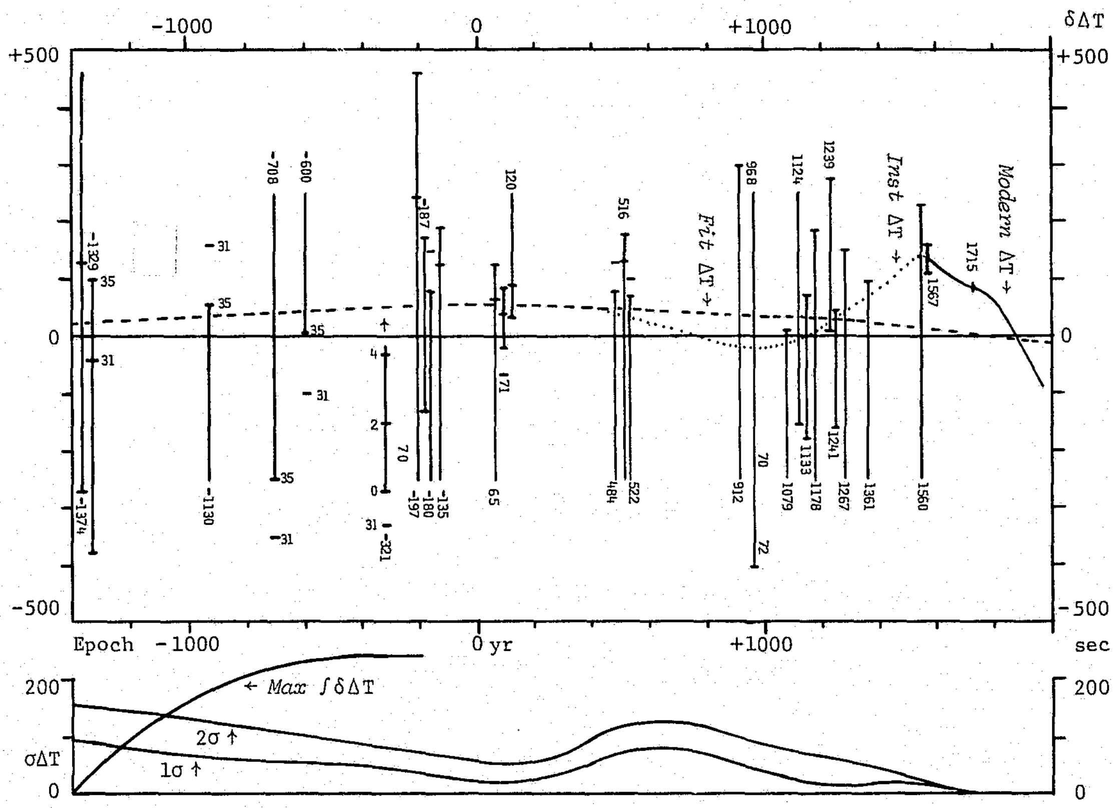
$$\delta\Delta T = \delta\Delta T_{nom} + \delta\dot{n}\,\frac{\partial\Delta T}{\partial\dot{n}} + \delta\ddot{\Omega}\,\frac{\partial\Delta T}{\partial\ddot{\Omega}} - \frac{\delta\dot{\Omega}}{.5T}\,\frac{\partial\Delta T}{\partial\dot{\Omega}}$$
$$- \left(\delta A + \delta B\cdot T + \delta C\cdot T^2\right) \qquad (37)$$

where the parameter changes are in the sense new value minus nominal ($\delta\dot{n} = \dot{n}_{new} - \dot{n}_{nom}$), and (37) can be used for both upper and lower limits as needed.

Figure 13 plots the $\delta\Delta T$ upper and lower limits for the observations of figure 9, mapped to $\dot{n} = -35$ and -31 "cy^{-2} via (37). The limits corresponding to these two accelerations are marked on those bounds which differ sufficiently (see the Key also). From this plot alone, or the equivalent calculation from figure 9 and (37), the determination of $\dot{n}$ and a complete ΔT polynomial can be made. The plot is $\delta\Delta T$ versus epoch of observation.

Figure 13

Observational Bounds



Key to Figure 13

ΔT range with limit.

ΔT limits at $\dot{n} = -31, -35$ "cy^{-2}.

Instantaneous (and Modern) ΔT.

1567 1130 Epoch (astronomical year) of the observation.

70 72 ΔT values from Newton (1970,1972) at $\dot{n} = -35$.

Final fit to mean ΔT polynomial.

Timed Contact, increasing delay: $\dot{n} = -35$; minutes.

Max $\int\delta\Delta T$ Maximum departure of ΔT instantaneous from mean between two epochs (read difference from abscissa scale above curve).

The solution space in ΔT expressed as a polynomial (18) is the set of all polynomials which can successfully pass between the limits shown in figure 13. Inconsistent observations (if any) are deleted, and medium/short term ΔT variations (few centuries time base) are smoothed. The solution space in $\dot{n}$ is defined as the range of $\dot{n}$ over which there exist polynomials for ΔT consistent with the data and the minimum deletion data set required by the filter. The solution for $\dot{n}$ is undertaken first.

Considering the curve "instantaneous ΔT" in figure 13 indicates that the only identifiable inconsistent equation is that from Klosterrath, 1133. Such a recent observation has no significant effect on the mean solution. Striking a rough mean of the modern data (since 1000) in figure 13, and turning attention to the ancient observations which dominate the overall determination of the accelerations, it is clear that there exist ΔT polynomials for the bounds computed at $\dot{n} = -35''\mathrm{cy}^{-2}$. It is also obvious that for $\dot{n}$ differing sufficiently from this value, one or more observation bounds will make it impossible to fit the ΔT polynomial between the limits. Each violated limit has a probability associated with it, and as we try to force $\dot{n}$ to wider values, thereby violating further limits, the probability that the given value of $\dot{n}$ is ruled out grows according to the ordinary laws of probability. To demonstrate the bounding of $\dot{n}$ it is only necessary to find the first few *pairs* of observations which stand as upper and lower bounds to the parameter.

The dashed curve in figure 13 labeled "*fit* ΔT" satisfies the observations at $\dot{n} = -35''\mathrm{cy}^{-2}$, and this is the place to begin in constructing the solution bounding pairs shown in Figure 14. At $-31''\mathrm{cy}^{-2}$ it is impossible to fit ΔT, and several observations operate against this value (for example 65 and 120). Computation reveals that the cutoff point for this pair is $\dot{n} = -31.9\ ''\mathrm{cy}^{-2}$, and this value is entered as a bound $\dot{n}_{bd}$ on line 3 of figure 14.

Figure 14 is constructed by finding the sequence of upper and lower bounding pairs as just discussed. The columns include several direct and useful numbers. [1] The observation pair number. [2] The observation pair epochs. [3] The difference between the appropriate UL and LL of figure 9 which expresses the "width" of the common area through which the ΔT polynomial can be passed, for $\dot{n} = -31''\mathrm{cy}^{-2}$. Figure 9 is computed for $-32.5''\mathrm{cy}^{-2}$ as noted in figure 10, but the node acceleration correction at this $\dot{n}$ from (14) and (9) gives a correction to the apparent $\dot{n}$ of about $+1.5\ ''\mathrm{cy}^{-2}$, and this is added into the solution at this point. [4] This is the difference of the partials $\partial\Delta T/\partial\dot{n}$ for the two observations in the pair from figure 9. [5] The bound is computed, therefore, from: $\dot{n}_{bd} = -31.0 - \delta t\ /\ \partial\Delta T$. Exact computation, and the need to account for the change in the nodal acceleration correction, makes the numbers given in figure 14 differ slightly from rounded values in figure 9.

Figure 14

Solution Bounding Pairs

#	Observations	δt	$\partial\Delta T$	$\dot{n}_{bd}$	$P_{\dot{n}}$	P_{inf}	$\dot{n}$ Estimate
Upper Bounds							
1	-135, 120	-35	30.2	-29.8	.98	.99	-36.7 ± 2.5
2	-1374, -1130	-30	77.5	-30.6	.40	.97	
3	65, 120	25	29.3	-31.9	.78	.96	-37.4 ± 2.7
4	-1374, -1329	170	141.	-32.2	.64	.82	-34.3 ± 1.9
5	71, 120	52	25.0	-33.1	.49	.49↑	-34.4 ± 2.0
Lower Bounds							
6	-1130, -600	260	54.4	-35.8	.40	.40↓	-34.4 ± 2.0
7	-600, -180	246	45.0	-36.5	.76	.78	-34.3 ± 1.9
8	-180, 71	208	29.2	-38.7	.47	.86	
9	-708, -180	496	41.8	-42.9	.85	.95	-37.4 ± 2.7
10	-1374, -1329	1770	141.	-43.4	.64	.98	
11	-1374, -708	1605	129.	-43.6	.72	.99	-36.7 ± 2.5

The probability associated with a given pair-bound is clearly the product of the truth probabilities for the two observations, since the inference fails if *either* is wrong:

$$P_{\dot{n}} = P(t)_i \cdot P(t)_j \tag{38}$$

In the example, $(.80 \times .98 \simeq .78)$ as indicated in the column $P_{\dot{n}}$ of figure 14. Other bounds are listed in monotonic order in figure 14.

Moving from the middle of figure 14, up and down through the upper and lower bounds, the probability that $\dot{n}$ itself has been bounded increases. As each successive bound is crossed, the individual probabilities for each bounding pair $P_{\dot{n}}$ will compound in yielding the overall inference probability P_{inf} that the actual value of $\dot{n}$ is within the solution space bounded by that value. If each bound arises from independent observations, then the probability that $\dot{n}$ is bounded inside the nth $P_{\dot{n}}$ will be:

$$P_{inf} = 1 - \prod_{i=1}^{n} (1 - P_{\dot{n}})_i \tag{39}$$

Reading up or down column P_{inf} in figure 14 from the middle will provide the running product (39). It will sometimes happen that a pair of bounding observations includes an observation which has already been included in a previous pair. It would be incorrect to include the same observation's probability of truth twice in (39). Consider two bounding pairs with three observations p, q, and r:

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p with q, and p with r. This pair of crossed bounds must be replaced with the single bound: p with (q *or* r) in evaluating (39), and this is reflected in figure 14. This process is much easier to do, than it is to explain.

The final column of figure 14 demonstrates the conversion, at last, of the probabilities into the more familiar solution with uncertainty. This can now be done without the devastating loss of sensitivity which would have accompanied the expression of equations of condition in least squares form. For any pairs of lines in figure 14 where P_{inf} values from upper and lower bounds are roughly equal, we can form a weighted mean solution for $\dot{n}$ (Estimate) from the $\dot{n}_{bd}$ corresponding to the P_{inf} lines. An approximate standard deviation of this mean can be computed from P_{inf} and the difference between the $\dot{n}_{bd}$ values, to make the probability roughly $^2/_3$ that the true value lies within the standard error. It is unnecessary to make this calculation with great precision, and an exact formula would be rather complex, and in any case, require assumptions about the Gaussian distribution.

Muller (1975) prefers the $\dot{n}$ estimates near P_{inf} probabilities between 1 and 2σ (*i.e.* .67 and .95), rather than the remote limits at the higher probabilities; nearer bounds to the crossover point in the middle of the figure are probably most significant. An $\dot{n} = -34.5 \pm 3$ "cy^{-2} was adopted, but values between −34.3 and −37.4 could be defended on various grounds. Deletion of the observations at 71 and −1130 would tend to move $\dot{n}$ nearer −35.5, indicating the small effect which arises in deleting these possibly controversial observations. There are other bounding pairs ready to enter the figure 14 at the top and bottom to replace deleted data from the table. It is unlikely that any defensible choice could move $\dot{n}$ more than the standard error. Applying the correction to $\dot{n}$ from $\dot{\Omega}$ (12), and retaining the Muller (1975) estimate from figure 14 of −34.5 "cy^{-2} we have from the large solar eclipses:

$$\dot{n}_{LSE} = -30.0 \pm 3.0 \text{ "cy}^{-2} \quad (40)$$

To have a consistent and accurate ephemeris, it is necessary to apply the above noted corrections to the ILE (1954) as follows:

$$\dot{\Omega} = +4.39 \text{ "cy}^{-1} \text{ and } \ddot{\Omega} = -0.120 \text{ "cy}^{-2} \quad (41)$$

Equations (40) and (41) if applied to the lunar theory will provide the best fit to the *ancient* solar eclipses.

The best fit to ΔT is indicated by the dashed curve in figure 13, and corresponds to:

$$\Delta T = 20 + 114 \cdot T + 38.30 \cdot T^2 \pm \sigma\Delta T = .3T^2 + 50 \quad (42)$$
$$@\ \dot{n}_t = -30.0 \text{ "cy}^{-2}$$

The uncertainty was estimated in Muller (1975) on the basis of deleting data and bending the ΔT polynomials, and applies to ΔT *given* $\dot{n}$.

The estimate of $\dot{\omega}/\omega$ comes from (42) and (17):

$$\dot{\omega}/\omega_{LSE} = -24.3 \pm 2.0 \times 10^{-11} \text{yr}^{-1} \quad (43)$$
$$\sigma\dot{\omega}/\omega = 0.2 \ (\textit{given } \dot{n})$$

The various linear combinations of the parameters can be formed to provide very useful conversion relationships. Since the A and B coefficients in (42) are relatively constant over $\dot{n}$, it is sufficiently accurate to express the linear relationship to C from (17) and (23):

$$\frac{\partial C}{\partial \dot{n}} = -.980 \quad (44)$$

The result (42) can be mapped to other values of $\dot{n}$ if desired. From (40), (42) and (44):

$$C = -.980\dot{n} + 8.90 \quad (45)$$

where (as elsewhere) the units are as stated above. For $\dot{\omega}/\omega$ given $\dot{n}$ we use (45) and (17):

$$\dot{\omega}/\omega = +.622\dot{n} - 5.65 \pm 0.2 \quad (46)$$

and $\dot{n}$ given $\dot{\omega}/\omega$ similarly:

$$\dot{n} = +1.61\dot{\omega}/\omega + 9.10 \pm 0.3 \quad (47)$$

Also from (17) and (42) we can obtain $\dot{\omega}/\omega$ given ΔT at an epoch T:

$$\dot{\omega}/\omega = .635 \left(\frac{20-\Delta T}{T^2} + \frac{114}{T} \pm \frac{\sigma\Delta T}{T^2} \right) \quad (48)$$

Constancy of the Accelerations

Newton (1970), page 644, concluded that the acceleration(s) changed substantially near epoch 700 and were far from constant during the historical period. He defined a parameter D" to measure the constancy of the linear relationship between $\dot{n}$ and $\dot{\omega}/\omega$ as follows:

$$D'' = \dot{n} - 1.6073\ \dot{\omega}/\omega \quad (49)$$

which is equivalent to equation (47) above, D" corresponding to the constant term, and we have:

$$D'' = +9.10 \pm 0.3 \text{ "cy}^{-2} \quad (50)$$

Newton found various values between ±20 from selected subsets of his data and concluded that the accelerations had varied. Muller (1975) shows that the typical short and medium term variations in ΔT such as those shown in figure 13 and as discussed by Morrison & Ward (1975) require the use of at least 800 year time bases in the estimation of whether the parameters have varied during historical time. Periods of 400 − 800 years are usable if errors larger than the parameter uncertainties (40), (43) are acceptable; less than 400 years will be quite unreliable in this connection.

Figure 15

Various Determinations of D"

#	Source	Epoch Range		D"
1	Newton 1970	-500	+500	+3.0†
2	Newton 1970	+500	1300	-6.1†
3	Newton 1970 corrected	-500	+500	+8.6
4	Newton 1970 corrected	+500	1300	+7.4
5	Newton 1970 p. 641	1650	1970	-18.0§
6	Muller 1975	1567	1975	+10.5
7	Muller 1975	various		+5 to +12
8	Muller 1975 & This paper	mean		+9.1

† ΔT modeling error - see text. $''cy^{-2}$

§ Insufficient data base. See note page 23 below.

Figure 15 summarizes the debate regarding constancy of the accelerations. Newton (1970) modeled ΔT according to (16) in computing $\dot{\omega}/\omega$ via (17). To examine what Newton (1970) would have gotten with the correct expression (18), it is necessary only to convert his $\dot{\omega}/\omega$ results back to a value of ΔT via (16), and plot them directly in ΔT. This was done in figure 13 near the years +1000 and -200 and marked "70" for Newton (1970). These ΔT values can now be converted correctly into $\dot{\omega}/\omega$ via (48), which when combined with the corresponding value of $\dot{n}$, yields D" from (49). The first two lines in figure 15 give Newton's D" solutions, the next two lines give the solutions corrected for modeling ΔT as above. Line 5 was his speculation on the modern value from occultation data. Muller (1975) examined the modern occultation data of Martin (1972) and Brouwer (1952), including the absolutely reliable eclipse observation by Clavius in 1567, and found the D" value on line 6; other subsets of the data (including solid and dotted curve in figure 13) provided a range of solutions as on line 7. The consistency between the correctly modeled determinations of D" in figure 15 proves that there has been no significant change in the relationship between the two accelerations during the historical period.

The question can be asked, would we fail to see a *proportional* variation in the accelerations when examining the constancy of D"? This test will indeed be weak if the accelerations vary together according to their mean partials (49). If the *tidal* couple were to change, as has been suggested by Newton (1970), then the parameters are related by the functions (19) and (20), which differ from (23) and a change in the tidal couple would be visible with substantial sensitivity. The medium term variations in ΔT, figure 13, bear further study.

I have just received a copy of Newton (1976), and therefore add this note in proof. Newton now accepts solving for constant and linear terms in (18), although he says that this must *not* be done by correcting the ephemeris. He fails to see that (18) and (3) are the same except for a constant scale factor, and it does not matter which is corrected. Although he still believes that the accelerations have changed, it is clear that the approach taken in Muller (1975) and above is now accepted by Newton. To quote from his page 12: "Similarly, when we deal with the acceleration of the sun, we should not alter the constant and linear terms in the ephemerides in response to an acceleration estimated from ancient data, since we have strong reasons to believe that the accelerations have not been constant since ancient times. Instead, we should estimate the constant and linear terms entirely from recent data. We should also estimate the accelerations from recent data and compare with the values estimated from ancient data." The approach taken in this paper is *precisely* what Newton suggests. It is clear from figures 13 and 14 that the ancient data alone determine the accelerations, the medieval-to-modern data span provides coefficient B, and the current epoch establishes A in (18). The solar system integration, Muller *et. al.* (1976), found that the constant and linear terms in the *lunar* ephemeris (1) do not require significant corrections. The entire scheme is therefore consistent and will correctly estimate the accelerations.

Measuring the constancy of the accelerations is a basic goal of the recent studies in this field, and it is important to be quite specific in clearing up the misconceptions. Newton (1976), pages 11 & 12, appears to concentrate on three points of evidence in support of his contention that the accelerations have changed during historical time. [1] Apparent changes in D" (or the equivalent) are indicated by ancient observations near epochs +700 and +1300. [2] The coefficients A and B in (18) must be solved from recent data, by which he sometimes seems to mean "not ancient" and at other times "very recent" *i.e.* the last 200 years. [3] He suggests comparing accelerations determined from ancient and modern (1700-1975) data. Each of these three points will be considered in turn. Newton's earlier works used D", equation (49), to test the constancy of the accelerations. Newton (1976) refers to ν_M', the lunar acceleration on solar time, but this differs from D" by only the small value of the solar acceleration ν_S' which is near -2.5 $''cy^{-2}$: $\nu_M' = D'' + \nu_S'$.

[1] Having given the quoted statement above, Newton (1976) still employs (16) instead of (18) in estimating $\dot{\omega}/\omega$, which in turn is central to D" and ν_M' in the test for constancy. Newton's conclusion that the accelerations changed near +700 and +1300 is based on the D" differences between the data sets noted in figure 15, lines 1 & 2. When his results are corrected, the ancient and medieval data show little variation, lines 3 & 4. This proves that it is Newton's reliance on equation (16) instead of (18)

in the presence of significant error in the rate of Newcomb's solar ephemeris (3) which gives rise to the apparent, but quite *spurious* change in the accelerations near these epochs which he notes. Another way to see the problem is to note that if (16) is to be used, we must have $T = 0$ be the epoch at which coefficient B in (18) is zero. As Muller (1975) notes, it can be seen that this occurs near the year 1800, not 1900 as assumed (in effect) by using equation (16) in Newton's works. See (62) and item [3] below, for examples. The solar ephemeris condition of equation (3) could be *mapped* to any year, say between 1770 and 1800, since the epoch is quite *arbitrary*. Had Newton used *that* epoch, he would have found the D" values given in figure 15 lines 3 and 4 instead of lines 1 and 2. Unfortunately, Newton (1976) notes that it is *all right* to solve for A and B in (18), but fails to apply the implications of what is in fact the *necessity* of doing so to his current and earlier results. He therefore persists, erroneously, in his belief that his earlier data analysis indicates a real change in the accelerations near +700 and +1300.

[2] As noted in the first paragraph of this note, the procedure of figures 13 and 14 determines coefficients C, B and A of equation (18) with ancient, medieval, and modern data respectively, and *independently*. If a determination of these coefficients is desired from a more recent data span, the recourse must be to the next paragraph.

[3] Newton (1976), page 12, says that Muller & Stephenson (1975) ignores the difference between D" lines 5 and 8 of figure 15 (in effect), thereby missing the change in the accelerations. Muller (1975) investigated this last vestige of Newton's argument by examining ΔT from 1567 - 1975 as follows: {a} Clavius' observation of a total solar eclipse in 1567, $\Delta T = +104 \pm 25s$; {b} Martin (1972) mean at epoch 1640, $\Delta T = +22 \pm 5$; {c} from 1650 - 1900, $\Delta T \simeq 0$; {d} ΔT then rising smoothly to +45s near 1975. Almost any D" can be obtained by fitting various data 1650-1975 as done by Newton. A representative 1567 - 1975 fit results from solving (18) from three points: 1567 (+104), 1654 (+2), and 1970 (+40). This provides $\Delta T = -24 + 68T + 32T^2$, and hence $\dot{\omega}/\omega = -20.4 \times 10^{-11}\, yr^{-1}$ from (17) corresponding to the ephemeris $\dot{n} = -22.44$ $''cy^{-2}$ used in the computation. From (49) we find $D'' = +10.5\ ''cy^{-2}$ as shown on line 6 of figure 15. Other data selections in this interval provide D" in adequate agreement with figure 15 lines 3, 4, and 8; not line 5. This is in better agreement, in fact, than we have any right to expect given the relationship between δT (the time base of the data in cy) and the uncertainty in D" arising from medium term ΔT variation, Muller (1975): 2cy, 19; 3cy, 13; 4cy, 8.8; 8cy, 3.9; 10cy, 2.4 $''cy^{-2}$. Newton overlooks this, and his final point vanishes.

For *all* data spans which *are* long enough, D" remains in very good agreement, observationally *demonstrating* that the accelerations have remained constant within uncertainties (about $\pm 3\ ''cy^{-2}$) corresponding to averages over 8 - 10 centuries.

The Earth's True Nontidal Acceleration

Under the assumption that $\dot{G}/G = 0$, we can solve for the earth's apparent nontidal rotational acceleration from (40) and (43) via (22):

$$\dot{\omega}/\omega_{ANT} = +\ 11.3 \pm 4 \times 10^{-11}\, yr^{-1} \tag{51}$$

This is about twice the currently suggested expectation from postglacial uplift, Kaula (private communication); see Dicke (1969) and O'Connell (1971). Obviously, if real rather than apparent, this acceleration must arise from changes in earth's moment of inertia or interchange of angular momentum within the earth, with a time-constant long compared with the historical period. Yukutake (1972) suggests that maintenance of the changing geomagnetic dipole moment might account for $+5 \times 10^{-11}\, yr^{-1}$. The current westward drift of the earth's magnetic field might be revealing the results of an accelerating couple inside the earth, but the required interchange of momentum demanded by 3000 years of this seems rather large. Lyttleton (1965) considers the possibly controversial idea that the earth's core is a phase change in mantle material, and makes some predictions of planetary conditions in the solar system (1969) which have proven interesting in view of recent observations by interplanetary spacecraft. With this hypothesis he resolves the Jeffrey's tidal inconsistency, and finds a consequent nontidal acceleration of the earth's rotation, Lyttleton (1976), of:

$$\dot{\omega}/\omega_{NT} = +\ 8 \times 10^{-11}\, yr^{-1} \tag{52}$$

This is in adequate agreement with (51), constituting another interesting prediction of the core phase change model. It appears that the *geophysical* theories are in a state of flux, no one of them yet proven. A significant $\dot{G}/G$ remains a possibility, however, see below.

Lambeck (1975) undertakes a detailed study of the theoretical earth-moon tides, and obtains reasonably good agreement with observations from his computation, finding: $\dot{n} = -35 \pm 4\ ''cy^{-2}$, figure 1.

Cosmology and Final Equations of Condition

The earliest suggestion that $\dot{G}/G$ could be observed in solar system astronomy is due to Dicke (1957). If G has a rate of change, then when observed on an invariant time scale such as Atomic Time (or the rotating earth), all planetary orbits expand and the periods lengthen proportionally:

$$2 \cdot \dot{G}/G = \dot{n}_a / n \tag{53}$$

where $\dot{n}_a$ is the orbital longitude acceleration observed on invariant time. In the Dirac (1973) cosmology, it is the atomic unit of length which is changing, but the observational principle remains

the same except for a sign ambiguity and scale factor change (the 2 in equation (53) becomes 1).

Dicke (1966) noted that if the earth's rotation can be calibrated for all geophysically induced torques, then it can be used as the cosmologically invariant time. Since the earth's observed rotational acceleration $\dot{\omega}/\omega$ is measured with respect to the sun's position, it will *appear* to be accelerated in the presence of $\dot{G}/G$:

$$\delta\dot{\omega}/\omega_{obs} = -\,2.0\ \dot{G}/G \qquad (54)$$

If the earth was a completely rigid body, only equations (54) and (22) combined with suitable observations, would be required to separate $\dot{\omega}/\omega_{TNT}$ (the true nontidal earth acceleration) from $\dot{G}/G$, thereby *observationally* determining both. This was undertaken in Muller (1975) with encouraging results. Two problems arose in this connection. The first, noted by Muller (1975), was to eliminate the necessity of solving for the lunar nodal rate $\dot{\Omega}$ from these observations. This problem is now resolved. The second problem arose in later analysis.

Murphy & Dicke (1964) estimated that as G decreases, the earth expands due to the relieved compressional stress provided by gravity according to the formula:

$$\delta\dot{\omega}/\omega_{obs} = +\,0.20\ \dot{G}/G \qquad (55)$$

This equation was reviewed with the same result by Nordtvedt & Will (1972). The bulk modulus of interior portions of the earth is known accurately from seismic wave observations, and the references found (55) on that basis. Lyttleton (private communication) has pointed out that the reference assumes the core is incompressible, uses a model for the mantle which is too stiff, and ignores the known phase change at the intra-mantle boundary. Whether one accepts Lyttleton's (1965) model of the core-mantle boundary as a phase change or not, it appears that the expansion ratio found by Nordtvedt & Will (1972) is too small. An exact solution to the problem appears to require some effort, but approximations indicate increases of between 2.5 and 5 in the coefficient of $\dot{G}/G$ given in (55). Pending a review of this expansion question, a parametric study of this will be substituted for (55). Define ε as the expansion coefficient and write:

$$\delta\dot{\omega}/\omega_{obs} = \varepsilon\cdot\dot{G}/G\ ;\quad 0.2 < \varepsilon < 1.0 \qquad (56)$$

where ε probably lies in the range shown.

To include the effects of $\dot{G}/G$ in the reconciliation of all known effects in the earth's rotation, where $\dot{\omega}/\omega_{TNT}$ is the true geophysical acceleration, we modify (22):

$$\dot{\omega}/\omega_{obs} = 1.1 + 1.224\dot{n}_t - (2.0-\varepsilon)\dot{G}/G + \dot{\omega}/\omega_{TNT} \qquad (57)$$

Dicke (1966) calibrated the earth's rotational acceleration by way of a detailed theoretical consideration of the known geophysical mechanisms, thereby finding $\dot{\omega}/\omega_{TNT}$. It is proposed here to *solve* for $\dot{n}_t$, $\dot{G}/G$, and $\dot{\omega}/\omega_{TNT}$ directly from the observations. The ancient eclipses even with the timed solar data above are not enough by themselves to successfully solve for the three parameters. The full observational equations of condition from all applicable astronomical data will be written, including the optional constraint $\dot{G}/G = 0$.

Figure 16

Equations of Condition

#	$\dot{n}_t$	$\dot{G}/G$	$\dot{\omega}/\omega_{TNT}$	Obs σ_{obs}	Wt
1	$1.000\ \dot{n}_t$	$-\ 0.17\ \dot{G}/G$		$= -30.0 \pm 3.0$	.111
2	$1.224\ \dot{n}_t$	$-(2.11-\varepsilon)\dot{G}/G$	$+\ 1.0\ \dot{\omega}/\omega_{TNT}$	$= -25.4 \pm 2.0$	.250
3	$1.224\ \dot{n}_t$	$-(2.00-\varepsilon)\dot{G}/G$	$+\ 1.0\ \dot{\omega}/\omega_{TNT}$	$= -24.9 \pm 2.3$	.190
4	$1.000\ \dot{n}_t$			$= -26.0 \pm 2.0$	.250
5	$1.000\ \dot{n}_t$	$+\ 3.47\ \dot{G}/G$		$= -36.0 \pm 5.0$	.040
6		$1.00\ \dot{G}/G$		$= 0.0$	∞
1a	$1.000\ \dot{n}_t$			$= -30.0 \pm 3.0$	.111
2a	$1.224\ \dot{n}_t$	$-\ 1.00\ \dot{G}/G$	$+\ 1.0\ \dot{\omega}/\omega_{TNT}$	$= -25.4 \pm 2.0$	.250
3a	$1.224\ \dot{n}_t$	$-\ 1.00\ \dot{G}/G$	$+\ 1.0\ \dot{\omega}/\omega_{TNT}$	$= -24.9 \pm 2.3$	.190
5a	$1.000\ \dot{n}_t$	$+\ 1.73\ \dot{G}/G$		$= -36.0 \pm 5.0$	.040
	$''cy^{-2}$	$10^{-11}\,yr^{-1}$	$10^{-11}\,yr^{-1}$		

Figure 16 displays the adopted equations of condition from various observation sets. The first block gives the equations of condition which hold in the presence of the so-called *primitive* cosmologies, including Hoyle & Narlikar (1974), Brans & Dicke (1961), Peebles & Dicke (1962) and others. The second block applies to the Dirac (1973) cosmology. The sign on the coefficient of $\dot{G}/G$ has been set in agreement with his multiplicative matter creation postulate, and a negative value solved for $\dot{G}/G$ will support this postulate, whereas a positive value would point to additive creation.

Line one of figure 16 comes from (40), with the small correction to apparent $\dot{n}$ arising from the effect of $\dot{G}/G$ on $\dot{\Omega}$ from (9) and (14). Two is (43) & (57), where $\dot{\omega}/\omega$ is also subject to $\dot{\Omega}$ through its tie to $\dot{n}$ as in (46). Line three is the timed solar data (35), also using (57). Line four is the result of Morrison & Ward (1975) who reworked the Spencer Jones transits of Mercury (modern data 1670-1975). As already noted, *timed* lunar data is insensitive to error in nodal rate.

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VanFlandern (1975) and (1976) approaches the problem directly by making observations on Atomic Time, where from (53) we can write:

$$\dot{n}_a / n = \dot{n}_t / n + 2 \cdot \dot{G}/G \tag{58}$$

His most recent value (1976) for the lunar occultations 1955-first quarter 1976 is:

$$\dot{n}_a = -\ 36.0 \pm 5.0\ ''\text{cy}^{-2} \tag{59}$$

which gives rise to line five of figure 16. Line six merely provides the constraint $\dot{G}/G = 0$, which will be used as an alternative cosmological assumption.

In the Dirac cosmology we must replace equations 1, 2, 3 and 5 as shown in the bottom of figure 16. Since it is the atomic unit of length which is changing in this cosmology, there is no *real* dynamical expansion of the planetary orbits, and there is no effect on the node or on earth expansion.

Planetary and lunar ranging data will probably yield estimates of parameters with usable sensitivity in determining $\dot{G}/G$ within a few years, but they are not quite there yet - see for example Reasenberg & Shapiro (1976). The paleontological data of Pannella (1972) can yield estimates of $\dot{\omega}/\omega$ and $\dot{n}_t$, Kaula & Harris (1975). It is my view that: [i] the uncertainties in the *recent* data (*ca.* -50×10^6 yr) are too large (they quote $\dot{n} = -58 \pm 15\ ''\text{cy}^{-2}$); the data sample is limited and a very small systematic error in the fossil counts will obviate this result; [ii] the tidal couple may have changed significantly in even 50my, and almost certainly has over 450my (at which epoch they quote $\dot{n} = -18 \pm 5$, error estimate mine). Newton (1968), figure 1, finds $\dot{n}_t$ from near-earth satellites, but I am concerned about the short data span and complexities in the theoretical modeling, which (if incomplete) will seriously impair the results. These potential observational results are not included in the regression, for the reasons stated.

Solutions from all Astronomical Data

Figure 17 provides the solutions from the equations of condition (figure 16) shown in column two. Line one is the so-called primitive cosmology, line two on Dirac's cosmology, line three assuming $\dot{G}/G = 0$. Lines four and five parameterize ε, (56) & (57), for the primitive cosmology. Line six solves the Dirac equations with the Hubble rate from extragalactic data (68) added, to get the final $\dot{\omega}/\omega_{TNT}$.

The value of $\dot{n}_t$ remains between -27.2 and -28.1 for all cosmological assumptions. The value from modern observations due to Morrison and Ward (1975) in figure 16 line 4 is probably less subject to a systematic error than the ancient eclipses (line 1) because the latter does depend on a smaller data sample than one might prefer. The solution consistency and weights on each data set seem reasonable, however, and the overall solution is

Figure 17

The Solutions

#	Eqns	$\dot{n}_t$	σ	$\dot{G}/G$	σ	$\dot{\omega}/\omega_{TNT}$	σ	ε
1	1 - 5	-27.4	1.6	-2.3	1.5	+4.0	4.1	0.2
2	1a-5a	-27.2	1.7	-5.1	3.0	+3.1	4.4	any
3	1 - 6	-28.1	1.6	0.0		+9.2	2.5	any
4	1 - 5	-27.4	1.6	-2.3	1.5	+4.7	3.8	0.5
5	1 - 5	-27.4	1.6	-2.3	1.5	+5.9	3.2	1.0
6	1a-5a†	-27.2	1.6	-5.6	0.7	+2.5	2.5	any
		$''\text{cy}^{-2}$		$10^{-11}\,\text{yr}^{-1}$		$10^{-11}\,\text{yr}^{-1}$		

†With eqn. (68) added.

probably the best choice for a final value:

$$\dot{n}_t = -\ 27.2 \pm 1.7\ ''\text{cy}^{-2} \tag{60}$$

The agreement between these determinations of $\dot{n}_t$, and the values of $\dot{\omega}/\omega$ from figure 16 line 2, dependent on the lunar acceleration, and line 3, independent of the moon, builds confidence in the elimination of the Spencer Jones anomaly, and in the constancy of the accelerations during the historical period (within uncertainties) as argued on other grounds above.

Only the linear combination of parameters found from the ancient data can effectively provide the apparent value of $\dot{\omega}/\omega$ *corresponding* to this $\dot{n}_t$, and from (60) via (46):

$$\dot{\omega}/\omega = -\ 22.6 \pm 1.1\ \{0.2\} \times 10^{-11}\,\text{yr}^{-1} \tag{61}$$

The parenthetical uncertainty applies if $\dot{n}_t$ is perfectly known. This result remains in very good agreement with the value from timed solar data independent of the moon (35).

To be of practical use in the computation of astronomical ephemerides during the historical period, equation (60) must be accompanied by the complete expression for ΔT, replacing the values on page 87 in the Explanatory Supplement (1961). Using (60), (42) and (44) we find:

$$\Delta T = 20 + 114 \cdot T + 35.55 \cdot T^2 \quad \pm\ \sigma\Delta T = .3T^2 + 50 \tag{62}$$

To have a consistent, and correct, lunar ephemeris, it is necessary to add two corrections to the lunar node expression coefficients in (2). First the correction to the nodal rate determined from a direct integration of the solar system:

$$\dot{\Omega} = +\ 4.39\ ''\text{cy}^{-1} \tag{63}$$

Second, the theoretical correction to nodal acceleration resulting from the tidally induced expansion of the lunar orbit from (60) & (14):

$$\ddot{\Omega} = -.10 \quad ''cy^{-2} \tag{64}$$

where ½ this value is added to the coefficient of T^2.

The ephemeris based on equations (60), (62), (63) and (64) will provide the best fit to all astronomical observations, ancient and modern. This will apply equally well to all observations made on Universal Time regardless of the cosmological assumption regarding $\dot{G}/G$ since (62) is the *observed apparent* ΔT, and net $\ddot{\Omega}$ corrections arising from $\dot{G}/G$ are very small.

The values of $\dot{\omega}/\omega_{TNT}$ fall in the range $+2.5 \pm 2.5$ to $+9.2 \pm 2.5$ for the various cosmological assumptions. These values are in the range which current theoretical analyses view as reasonable, and it appears that there is no serious observational versus theoretical discrepancy - a significant improvement on results such as Newton (1970) who obtained $+23 \times 10^{-11} yr^{-1}$. The issue now appears to be clearly drawn. If there is a real or apparent $\dot{G}/G$, then there is *no* significant $\dot{\omega}/\omega_{TNT}$. To geophysicists committed to one or more of the several theoretical explanations for a significant nontidal earth acceleration, this may possibly cause *them* to believe that $\dot{G}/G = 0$! Conversely, most cosmologists will probably argue that *everything* is tidy and consistent - a reasonable $\dot{G}/G$ and an earth's tidal rotation satisfying the conservation of angular momentum! There is always the possibility of a bit of both, as Urey frequently and wisely points out. In this case, we would need a cosmology demanding a small fraction of the Hubble Constant as its rate (see below), and I am not aware of one in the current literature. It will be apparent below that present cosmologies are either consistent with the $\dot{G}/G$ found from the observations, or demand *more* $\dot{G}/G$ and a *negative* $\dot{\omega}/\omega_{TNT}$. This situation appears to be fairly strictly *either-or*. In view of this, it is illustrative to view the history of geophysical papers requiring some nontidal earth acceleration.

Munk & MacDonald (1960) and Dicke (1966), (1969) did not appeal to large nontidal accelerations in their theoretical analyses. The apparent *observational* values cried out for explanation, and several suggestions (see above) have been found by theoreticians. It may be unfair to suggest this, but the feeling persists that if it were suddenly possible to show *observationally* that $\dot{\omega}/\omega_{TNT} \simeq 0$, the theoretical analyses would soon follow to show that yes, after all, the earth-moon system tides do show apparent conservation of angular momentum. Against this we have several cosmologies, each of which demands *at least* as much apparent nontidal $\dot{\omega}/\omega$ (as $\dot{G}/G$) as the data now provide. These cosmologies were not constructed in response to the nontidal acceleration anomaly (as were the geophysical analyses), and yet the need from all of them is for enough $\dot{G}/G$ to easily replace all the $\dot{\omega}/\omega_{TNT}$ which the data can muster. This is too much of a common demand to dismiss as coincidence, and suggests that $\dot{G}/G \neq 0$ is more likely than a significant $\dot{\omega}/\omega_{TNT}$.

Rate of Change of G

Primitive cosmologies, for all values of ε, provide from figure 17:

$$\dot{G}/G = -2.3 \pm 1.5 \times 10^{-11} yr^{-1} \tag{65}$$

Theories consistent with the figure 16 equations of condition include: Hoyle & Narlikar (1974), Dicke (1957) & (1962), Peebles & Dicke (1962), providing:

$$H = \tfrac{1}{2}\tau^{-1}; \quad G \propto \tau^{-\frac{1}{2}}; \quad \dot{G}/G = -\tfrac{1}{2}\tau^{-1} = -H \tag{66}$$

where τ is the parametric age of the Universe, and H is the Hubble Constant. Sandage & Tammann (1975) obtain a Hubble Constant of:

$$H = 55 \pm 7 \text{ km/s/Mpc} \tag{67}$$

which implies a $\dot{G}/G$ from extragalactic observation:

$$\dot{G}/G = -5.6 \pm 0.7 \times 10^{-11} yr^{-1} \tag{68}$$

Equation (65) is over two standard deviations from the requirement of (68) and this is not good news for this class of cosmologies. On the other hand, the Brans & Dicke (1961) cosmology involves several parameters, and its implications probably bound $\dot{G}/G$:

$$0 \geq \dot{G}/G \geq -3H/(w+2) \geq -5 \times 10^{-11} yr^{-1} \tag{69}$$

with preferred values near $-2 \times 10^{-11} yr^{-1}$; see Weinberg (1972) page 629. The result (65) is in adequate agreement with this theory. I am not competent to judge the roots of theoretical cosmology, but have been frequently advised that the entire class of primitive cosmologies is in deep trouble on other observational and theoretical considerations. No doubt there are those who disagree, so suffice it to say, the numerical results of this paper cast some observational doubt on the primitive cosmologies *except* Brans-Dicke.

Dirac's cosmology also demands that $\dot{G}/G = -H$ and (68), which is in good agreement with the solution from figure 17 line 2 on this cosmology:

$$\dot{G}/G = -5.1 \pm 3.0 \times 10^{-11} yr^{-1} \tag{70}$$

The equations of condition implied by this cosmology are probably the best we have available, and this result is in good agreement with the finding of VanFlandern (1976), equations 4 and 5a of figure 16 alone:

$$\dot{G}/G = -5.8 \pm 3.1 \times 10^{-11} yr^{-1} \tag{71}$$

There are, in effect, two independent experimental ways to find $\dot{G}/G$ represented in figure 16 equations of condition. One is given in (71). The other is to *assume* $\dot{\omega}/\omega_{TNT} = 0$ and use equations 3a and 4 from figure 16. This uses the same source of $\dot{n}_L$ as the reference, but an entirely independent source for

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$\dot{\omega}/\omega$ which is also independent of the moon. Solving these equations yields:

$$\dot{G}/G = -6.9 \pm 3.0 \times 10^{-11}\,yr^{-1} \tag{72}$$

It is difficult to ignore the patterns of strong agreement between various determinations of $\dot{G}/G$: [1] from two independent lines of astronomical data from the solar system; [2] with the Hubble Constant determined from extragalactic data via the equations of the best available cosmology. [3] The solution for $\dot{G}/G$ with the Hubble constraint (68) on *any* current cosmology eliminates all significant nontidal earth acceleration $\dot{\omega}/\omega_{TNT}$. [4] No geophysically assumed $\dot{\omega}/\omega_{TNT}$ can eliminate the result (71) because VanFlandern's data is independent of the earth's rotation.

There is only *one* interpretation which will satisfy all of the observations, and that is to simply accept the cosmological $\dot{G}/G$ *solved* from the data. If the alternative of a large nontidal earth acceleration $\dot{\omega}/\omega_{TNT}$ is to be forced, *both* of the following must hold. [a] The VanFlandern (1976) observation must be biased enough to obviate his result. His analysis is a very difficult undertaking, and he has had problems in eliminating systematic errors: compare VanFlandern (1975) and (1976). His latest results are very encouraging, and the solution uncertainties are now declining very rapidly because a full 21 year data span is breaking correlations resulting from the 18.6 year lunar nodal rate. His latest results appear very strong, and the numerical values have not changed significantly in the last year of analysis. [b] The *true* cosmology must require a zero or very small $\dot{G}/G$ compared with the Hubble rate. Every modern cosmology has established this relationship at an equal or greater level than that required to completely eliminate the observed $\dot{\omega}/\omega_{TNT}$ as noted above. The Dirac cosmology may not be the final answer to every demand, but neither this nor any other specific cosmology needs to be adopted in order for this argument to follow. A cosmology intrinsically different from all of the recent suggestions must be advanced and justified before this constraint can be avoided.

The acceptance of the cosmological $\dot{G}/G$ found here and in VanFlandern (1976) is consistent with all of the observations. To accept the alternative geophysical $\dot{\omega}/\omega_{TNT}$ it is necessary to abandon a basic observation set (Atomic Time lunar occultations), as well as the basic thrust of every major cosmology (the relation of $\dot{G}/G$ to the Hubble Constant). This appears to be an unlikely, though perhaps not quite impossible, choice. Accepting the Hubble rate determination (68) as a further equation of condition added to figure 16 equations 1a - 5a probably provides the best available observational estimate of $\dot{\omega}/\omega_{TNT}$:

$$\dot{\omega}/\omega_{TNT} = +2.5 \pm 2.5 \times 10^{-11}\,yr^{-1} \tag{73}$$

The uncertainty of figure 17 line 2 is reduced by adopting the additional equation (68) instead of solving for all three parameters on the Dirac equations of condition and solar system observables only. The larger uncertainty can be taken if the inclusion of extragalactic data is not desired, as can the solution of line 1 if the primitive cosmologies are preferred. In any case, the $\dot{\omega}/\omega_{TNT}$ is consistent with zero, and there remains no need to find a geophysical explanation of any significant nontidal earth acceleration.

It is concluded that there are strong reasons for accepting the cosmological $\dot{G}/G$ found from the observations, and consistent with the Hubble Constant. The alternative of a large nontidal acceleration of the earth's rotation is unlikely, and the earth-moon system tides appear to conserve angular momentum wholly within the tidal couple to about $3 \times 10^{-11}\,yr^{-1}$ or better. The tidal accelerations are observed to remain constant within uncertainties during the historical period.

Summary of Principal Results

- Tidal accelerations constant within standard error
- Strong independent support for cosmological $\dot{G}/G$
- Nontidal earth acceleration consistent with $\dot{G}$
- Consistent with Dirac and Brans-Dicke cosmologies
- Ancient, modern, and extragalactic data agree

Figure 18

Principal Numerical Results

ANCIENT OBSERVATION RESULTS

$\dot{n}_t = -30.0 \pm 3.0$ "cy^{-2}	Eclipses Only
$\dot{\omega}/\omega = -24.3 \pm 2.0 \times 10^{-11}\,yr^{-1}$	Eclipses Only
$\dot{\omega}/\omega = -23.8 \pm 2.3 \times 10^{-11}\,yr^{-1}$	Timed Equinoxes
$D'' = \dot{n} - 1.61\dot{\omega}/\omega = +9.10 \pm 0.3$ "cy^{-2}	

RESULTS FROM ALL ASTRONOMICAL DATA

$\dot{n}_t = -27.2 \pm 1.7$ "cy^{-2}	
$\dot{\omega}/\omega = -22.6 \pm 1.1 \times 10^{-11}\,yr^{-1}$	
$\Delta T = 20 + 114 \cdot T + 35.55 \cdot T^2 \quad \pm\sigma\ \Delta T = .3T^2 + 50$	
$\dot{\Omega} = +4.39$ "cy^{-1} $\quad \ddot{\Omega} = -.10$ "cy^{-2}	
$\dot{G}/G = -2.3 \pm 1.5 \times 10^{-11}\,yr^{-1}$	Primitive Cosmology
$\dot{G}/G = -5.1 \pm 3.0 \times 10^{-11}\,yr^{-1}$	Dirac Cosmology
$\dot{\omega}/\omega_{TNT} = +2.5 \pm 2.5$ TO $+9.2 \pm 2.5 \times 10^{-11}\,yr^{-1}$	
↑Solve $\dot{G}/G$↑ ↑ $\dot{G}/G = 0$ ↑	

Glossary of Terms and Notation†

Term	Definition	Units
-762	Astronomical year: 1=1AD; 0=1BC; -1=2BC ...	
AT	Atomic Time: physical time kept by electron transitions in atoms, *e.g.* Cesium.	
ET	Ephemeris Time: Newtonian or Gravitational time kept by orbiting bodies.	
ABC	Coefficients in expression for ΔT (18)	seconds
cy	centuries	10^2 yr
D"	Newton's linear relationship between the accelerations (49)	$''\mathrm{cy}^{-2}$
G	Gravitational Constant; rate = $\dot{G}/G$	$10^{-11}\mathrm{yr}^{-1}$
H	Hubble Constant (Universe expansion)	km/s/Mpc
I	Inclination of lunar orbit to ecliptic	°
my	Million years	10^6 yr
P(t)	Truth probability of an observation	- -
T	Time from 1900 in cy.	cy
w	Parameter in Brans-Dicke cosmology	- -
$\dot{n}$	Lunar acceleration in longitude	$''\mathrm{cy}^{-2}$
$\dot{n}_t$	Tidally caused part of $\dot{n}$	$''\mathrm{cy}^{-2}$
$\dot{n}_a$	Apparent $\dot{n}$ observed on AT	$''\mathrm{cy}^{-2}$
$\dot{n}_s$	Apparent acceleration of the sun	$''\mathrm{cy}^{-2}$
n	A mean orbital rate in longitude	$''\mathrm{cy}^{-1}$
$\dot{\omega}/\omega$	Earth's rotational acceleration scaled by the mean rate	$10^{-11}\mathrm{yr}^{-1}$
ANT	Apparent nontidal (subscript $\dot{\omega}/\omega$)	- -
TNT	True nontidal (subscript $\dot{\omega}/\omega$)	- -
$\dot{\Omega}$	Correction to lunar nodal rate	$''\mathrm{cy}^{-1}$
$\ddot{\Omega}$	Correction to lunar nodal acceleration	$''\mathrm{cy}^{-2}$
ΔT	ET - UT	seconds
$\Delta\phi$	Change in latitude at conjunction	"
$\Delta\lambda$	Change in longitude at conjunction	"
$\Delta\Omega$	Change in nodal longitude	"
σ	Standard deviation of parameter uncertainty	
∂	Partial derivative	
Φ	Observed phase in an eclipse *e.g.* total	

† Astronomical terms not listed here may be found in the Explanatory Supplement (1961).

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